

# **ARYAN SCHOOL OF ENGINEERING & TECHNOLOGY**

**BARAKUDA, PANCHAGAON, BHUBANESWAR, KHORDHA-752050**



## **LECTURE NOTE**

**SUBJECT NAME- ENERGY CONVERSION-I**

**BRANCH – ELECTRICAL ENGINEERING**

**SEMESTER – 4<sup>TH</sup> SEM**

**ACADEMIC SESSION - 2022-23**

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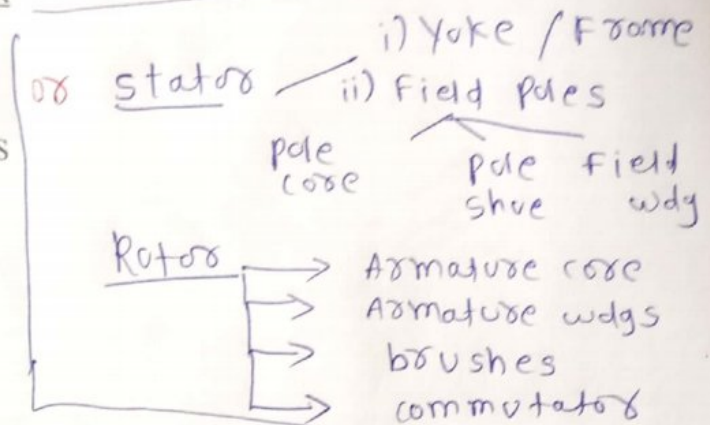
# INTRODUCTION:

It is an electrical machine which converts mechanical energy in to electrical energy .

## CONSTRUCTION OF D.C. GENERATOR

It consist of the following main parts .

- A. YOKE OR MAGNETIC FRAME
- B. POLE CORES AND POLE SHOES
- C. FIELD WINDING
- D. ARMATURE
- E. COMMUTATOR
- F. BRUSH



### A. Yoke or Magnetic Frame

- It is made of cost iron. ↙ small machine  
→ cast steel (large machine)
- It's function is to protect the inner parts of the machine from mechanical injury. ∴ it acts as protective covers for entire m/c.
- It holds the inner parts of the machine.
- It will not allow the magnetic flux which is produced in the field poles to go out.

### B. Pole Cores And Pole Shoes

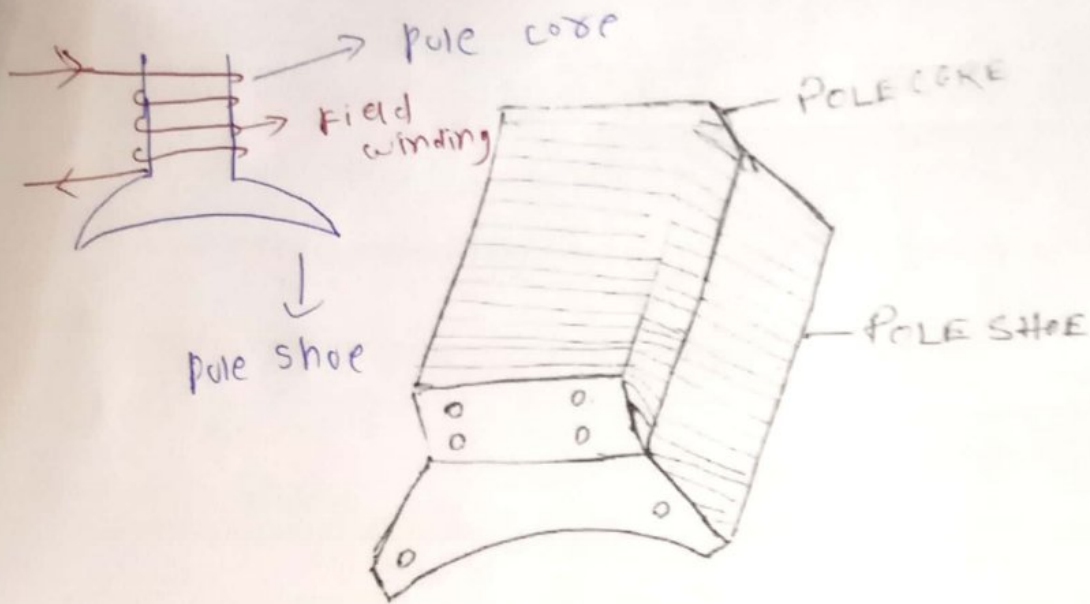
It is made of laminated silicon steel material . The order of lamination is 0.35mm to 0.5mm . The shape of pole shoes as shown in figure . They spread out the flux on the air gap and also being of larger crossection to reduce the reluctance of the magnetic path . so that the flux produced by the poles will be more...

pole core : cast steel

- i) It carries field winding & provides mechanical support to the pole shoe .
- ii) It acts as electromagnet when the field windings are excited .

pole shoe : cast steel laminations

- i) It distributes magnetic field uniformly in the air gap .



Silicon steel material is used to reduce the hysteresis loss . The support the field coils .

### c. Field Winding

It consists of thick copper wires wound over the pole cores .

When D.C current passes through than the electromagnetic is converted in to magnet and it will produce necessary magnetic flux .

### D. Armature

→ It is the rotating part of the machine .

It is made of laminated silicon steel material in cylindrical shape . The lamination are approximately 0.5mm thick .

There are so many slots in its outer periphery where the armature winding are placed .

### f. Commutator

→ It is made of hard-drawn copper material .

→ It is in cylindrical shape .

imp → The function of commutator is to collect the current from the armature conductor and convert the alternating current which is induced in the armature in to unidirectional D.C current .



→ The no. of commutator segments is equal to the no. of coils. Each commutator segment is separated from another by the help of mica insulation.

### BRUSH

→ It is made of carbon due to its (-)ve temperature co-efficient of resistance property.

emf, → It slip over the commutator and its function is to collect current from the commutator and supply to the external load circuit.

### WORKING PRINCIPLE

→ D.C Generator works according to the principle of Faraday's laws of electromagnetic induction.

→ When ever a conductor cuts the magnetic lines of force an emf is induced in it. Here the mechanical power is utilized to rotate the armature. The armature cut and the magnetic field an emf is induced on the armature conductors. The induced emf is

$$e = -N \frac{d\phi}{dt}$$

### TYPES OF ARMATURE WINDING

There are two types of armature windings.

- A. LAP WINDING ✓
- B. WAVE WINDING ✓

#### LAP WINDING

→ In case of lap winding the no. of poles is equal to no. of parallel paths. ( $A=P$ )

→ It is used where high current and low voltage is required.

$$A = P$$

#### WAVE WINDING

In case of wave winding the no. of parallel paths is always equal to two ( $A=2$ ).

It is used where high voltage and low current is required.

$$A = 2$$

### EMF EQUATION OF DC GENERATOR

Let  $P$  = No of poles

$\phi$  = Flux per pole in weber



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 $Z$  = Total no. of conductor

$N$  = Speed of armature in r. p.m

$A$  = No. of parallel paths

$\frac{Z}{A}$  = Number of conductors / parallel paths

The emf induced in the armature due to flux linkage in the conductor is given

By  $e = -N \frac{d\phi}{dt}$

Emf induced per conductor

$e = \frac{d\phi}{dt}$  ( $\therefore N = 1$ )

Now flux cut per conductor in one revolution.  $d\phi = p\phi$   $d\phi = P\phi$

$N$  = Number of rotation per minute.

Number of rotation per second =  $\frac{N}{60}$

Time taken to complete one revolution

$dt = \frac{1}{N/60} = 60/N$

$dt = \frac{1}{(N/60)} = \frac{60}{N}$

Now emf generated per conductor

$$e = \frac{d\phi}{dt}$$
$$= \frac{P\phi}{60/N} = \frac{P\phi N}{60}$$

$$e = \frac{d\phi}{dt} = \frac{P\phi}{(60/N)}$$
$$= \frac{P\phi N}{60}$$

Emf induced per parallel path

$$= \frac{P\phi N}{60} \times \frac{Z}{A} = \frac{P\phi Z N}{60A}$$

$$e = \frac{P\phi N}{60} \times \frac{Z}{A}$$
$$= \frac{P\phi Z N}{60A}$$

Generated emf ( $E_g$ ) =  $\frac{P\phi Z N}{60A}$

Le  $E_g = \frac{P\phi Z N}{60A}$

$$E_g = \frac{P\phi Z N}{60A} \text{ volts}$$

$\rightarrow$  For lap,  $A = P$ ,  $E_g = \frac{\phi Z N}{60}$  volts

$\rightarrow$  For wave,  $A = 2$ ,  $E_g = \frac{P\phi Z N}{120}$  volts.

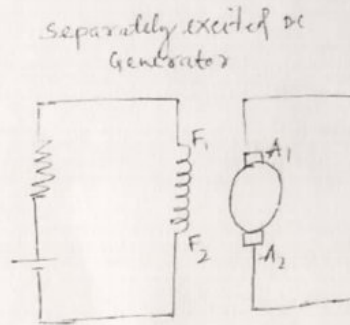
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## CLASSIFICATION OF D.C GENERATOR

D.C Generator are classified in to two types according to their excitation .

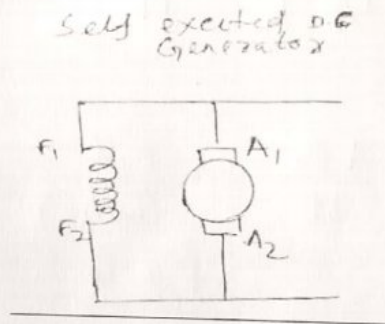
- A. SEPARATLY-EXCITED D.C GENERATOR
- B. SELF-EXCITED D.C GENERATOR

### SEPARATLY-EXCITED D.C GENERATOR



If the field winding is excited by some external independent dc source then it is known as separately excited D.C generator .

### SELF-EXCITED D.C GENERATOR

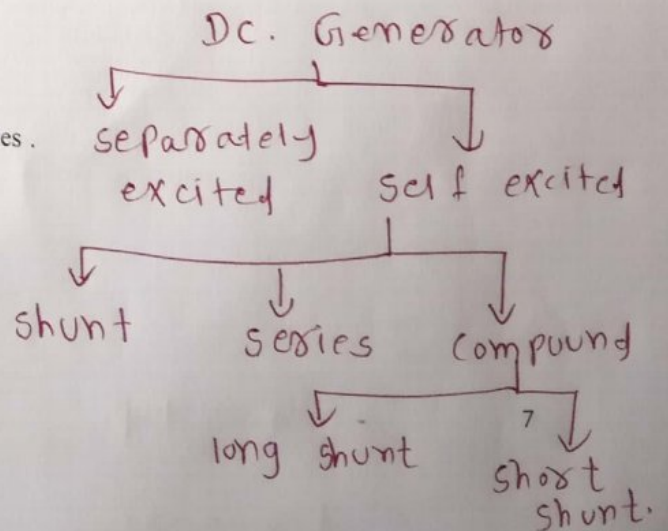


If the field magnets are excited by its own current , then it is known as self excited D.C Generator . It does not require any external source .

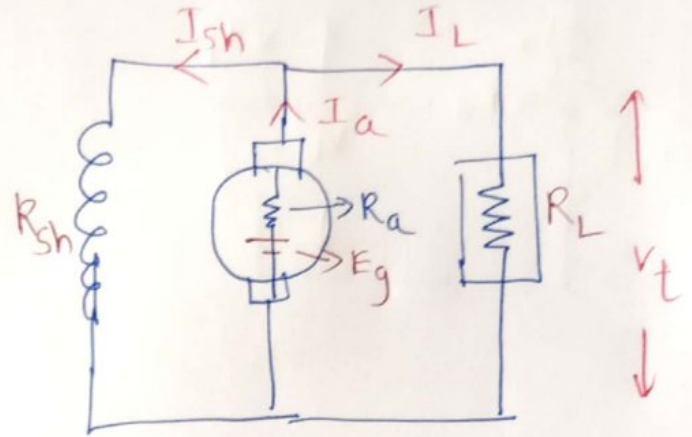
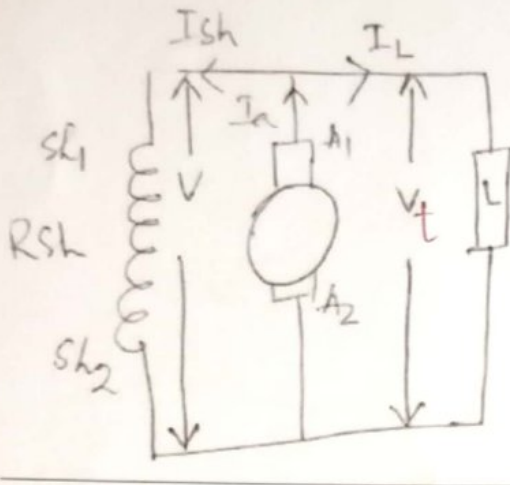
According to the connection of the field .

Winding Self-excited generators are classified in to 3 types .

- A. D.C Shunt Generator
- B. D.C Series generator
- C. D.C Compound Generator



DC Shunt Generator



The field winding is connected in parallel with the armature. The field winding is excited by the terminal voltage.

~~ESD~~

$$I_{sh} = \frac{V_t}{R_{sh}}$$

$$I_a = I_L + I_{sh}$$

Where  $V_t$  = Terminal voltage or voltage across the load.

$R_{sh}$  = shunt field resistance

$$I_a = I_{sh} + I_L$$

$$E_g = I_a R_a - B \cdot D = V_t$$

$$\text{or } E_g = V_t + I_a R_a + B \cdot D$$

$$E_g = V + I_a R_a + b \cdot d$$

Where  $R_a$  = Armature resistance which is very very small.

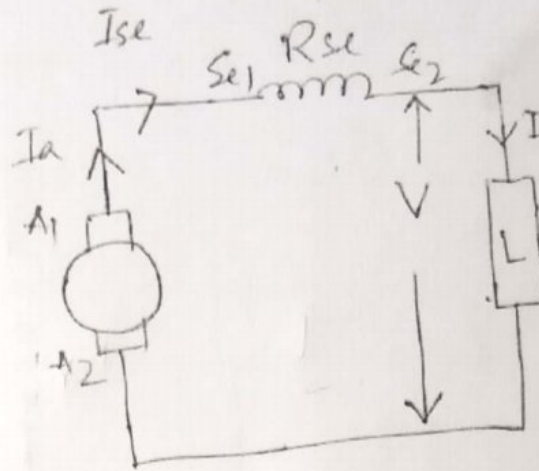
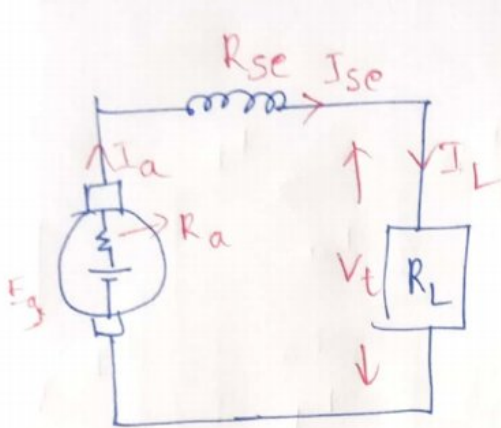
$I_a R_a$  = Armature Resistance drop

$B \cdot d$  = Brush contact drop

$E_g$  = Generated emf in the armature.



## D.C SERIES GENERATOR



DC Series Generator.

The field winding is connected in series with the armature .

Here  $I_a = I_{se}$

here

$$I_a = I_{se} = I_L$$

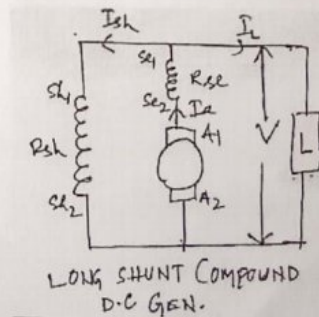
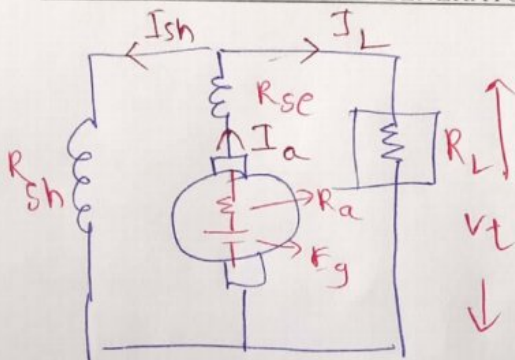
$$E_g = V + I_a(R_a + R_{se}) + B.D$$

Here the field is excited by the load current .

## D.C COMPOUND GENERATOR

It is the combination of series of field and shunt field .

### LONG-SHUNT COMPOUND GENERATOR



LONG SHUNT COMPOUND D.C GEN.

$$I_{sh} = \frac{V}{R_{sh}}$$

$$I_a = I + I_{sh}$$

$$E_g = V + I_a R_a + I_a R_{se} + B.D$$

$$I_{sh} = \frac{V_t}{R_{sh}}$$

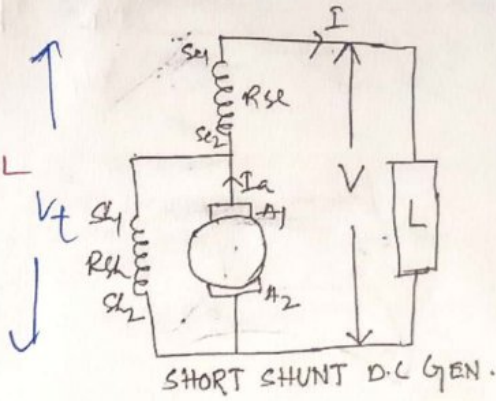
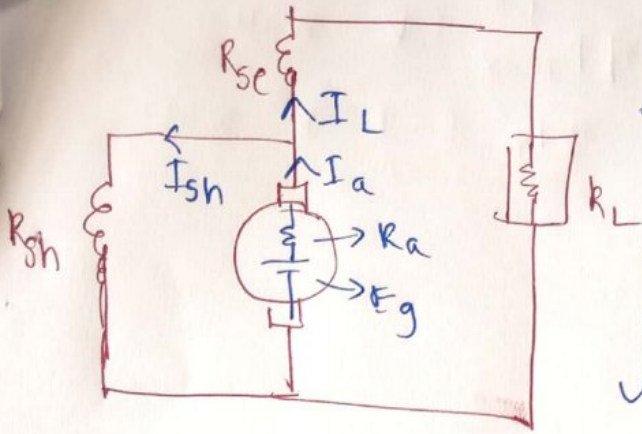
$$I_a = I_{sh} + I_L$$

$$E_g - I_a R_a - I_a R_{se} - B.D = V_t$$

$$\Rightarrow E_g = I_a R_a + I_a R_{se} + B.D + V_t$$

## SHORT-SHUNT COMPOUND GENERATOR

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Here  $I_{se} = I$

$$I_{sh} = \frac{V + I_{se}R_{se}}{R_{sh}}$$

$$I_a = I + I_{sh}$$

$$E_g = V + I_a R_a + I R_{se} + b.d$$

$$I_L + I_{sh} = I_a$$

$$E_g - I_a R_a = V \quad \frac{V}{R_{sh}} = I_{sh}$$

$$V - I_L R_{se} = V_t$$

- A. Cumulatively Compound dc generator .
- B. Differentially Compound D.C generator .

### LOSSES IN A.D.C MACHINE .

There are 3 types of losses in a D.C machine .

- A. Copper loss =  $I^2 R$
- B. Iron / Core / Magnetic Loss
- C. Mechanical loss

#### COPPER LOSS (30-40%)

The loss occurs due to the resistance . It is about 30-40%

- i. Armature copper loss =  $I_a^2 R_a$  ✓  $I_a^2 R_a$
- ii. Series field copper loss =  $I_{se}^2 R_{se}$   $I_{se}^2 R_{se}$
- iii. Shunt field copper loss =  $I_{sh}^2 R_{sh}$   $I_{sh}^2 R_{sh}$

#### IRON / CORE / MAGNETIC LOSS (20-30%)

The losses occur in the machine armature and field core .

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It consists of

- i. Hysteresis loss =  $W_h = h k B_{max}^{1.6} f V$  wats  
ii. Eddy current loss =  $W_e = k B_{max}^2 f^2 t^2 v$  wats

$$W_h = \eta B_{max}^{1.6} f V \text{ Watts}$$
$$W_e = \eta B_m^2 f^2 t^2 v^2 \text{ Watts.}$$

Where  $h$  = steinmertz co-efficient of hysteresis constant .

$B_{max}$  = Max . Flux density in  $wb/m^2$

$f$  = Frequency in HZ

$t$  = Thickness of lamination

$V$  = Volume of material

For reducing hysteresis loss silicon steel material is preferred .

For reducing eddy current loss laminated sheets are used . lamination is done in order to reduce eddy current loss.

### MECHANICAL LOSS (10-20%)

It consists of friction and windage loss of rotating machine . air shunt .

### STRAY LOSS

It is the sum of iron loss and mechanical loss stray loss = Iron loss + Mechanical loss

### CONSTANT LOSS ( $W_c$ )

It is the sum of stray loss and shunt field copper loss .

$$W_c (\text{Constant Loss}) = \text{stray loss} + \text{shunt field copper loss}$$

$$= \text{Iron loss} + \text{mechanical loss} + \text{shunt field copper loss}$$

Since shunt field current is constant the shunt field copper loss is also constant .

### EFFICIENCY OF A D.C MACHINE

Efficiency is defined as the ratio of out put to input of a machine

$$\text{Efficiency } (\eta) = \frac{\text{out put}}{\text{input}}$$

$$= \frac{\text{out put}}{\text{out put} + \text{loss}}$$

$$\text{Efficiency } (\eta) = \frac{O/P}{I/P} = \frac{O/P}{O/P + \text{losses}}$$



## CONDITION FOR MAXIMUM EFFICIENCY OF D.C GENERATOR

$$\text{Efficiency} = \frac{o/p}{i/p} = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$= \frac{o/p}{o/p + W_c + I_a^2 R_a}$$

$$= \frac{VI}{VI + W_c + I_a^2 R} \quad (\because I_a \gg I)$$

$$= \frac{VI}{VI + W_c + I^2 R_a} \quad (\because I_a = I)$$

The efficiency will be maximum when  $\frac{d\eta}{dI} = 0$

$$\Rightarrow \frac{d}{dI} \left( \frac{VI}{VI + W_c + I^2 R_a} \right) = 0$$

$$\Rightarrow \frac{v[vI + W_c + I^2 R_a] - vI[v + 2IR_a]}{(VI + W_c + I^2 R_a)^2} = 0$$

$$\Rightarrow v[vI + W_c + I^2 R_a] - v^2 I + 2vIR_a = 0$$

$$\Rightarrow v[vI + W_c + I^2 R_a] - vI[v + 2IR_a] = 0$$

$$\Rightarrow [vI + W_c + I^2 R_a] = I[v + 2IR_a]$$

$$\Rightarrow vI + W_c + I^2 R_a = vI + 2I^2 R_a$$

$$\Rightarrow W_c - I^2 R_a + 2I^2 R_a$$

$$\Rightarrow W_c = I^2 R_a$$

Efficiency will be maximum when constant loss is equals to variable loss.

The load current corresponding to maximum efficiency is given by  $I = \sqrt{\frac{W_c}{R_a}}$

$$I = \sqrt{\frac{W_c}{R_a}}$$

$$\text{efficiency} = \frac{o/p}{o/p + \text{losses}}$$

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$$= \frac{o/p}{o/p + W_c + I_a^2 R_a}$$

$$= \frac{VI}{VI + W_c + I_a^2 R}$$

$$= \frac{VI}{VI + W_c + I_a^2 R_a}$$

$$\frac{d\eta}{dI} = 0$$

$$\Rightarrow \boxed{W_c = I^2 R_a}$$

## ARMATURE REACTION

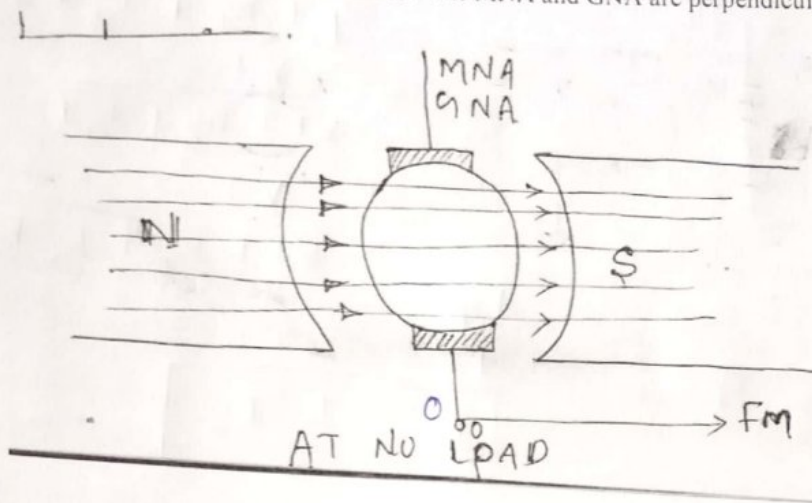
When current flows through the armature conductors a magnetic field is produced.

This magnetic field due to armature current weakens and distorts the main magnetic field produced by the field poles. This effect is known as armature reaction.

### AT NO-LOAD

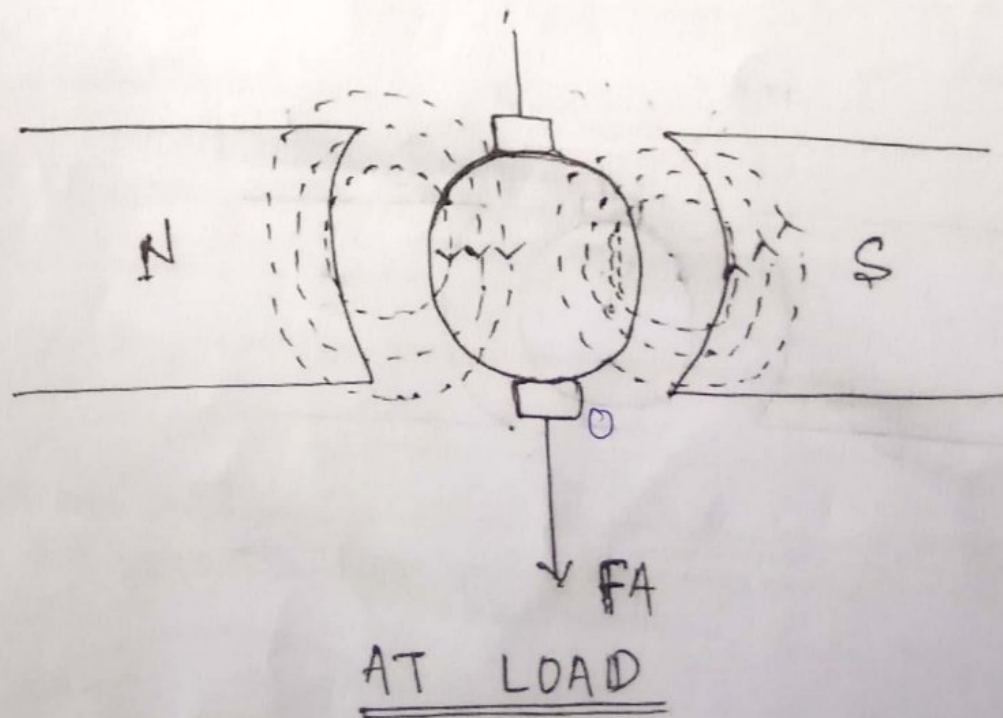
The armature current is zero or small volume. This is due to the field flux. The vector of  $F_m$  represents the MMF producing the main field. Here MNA (Magnetic Natural Axis) and GNA (Geometrical Natural Axis) are co-incident with each other. The MNA and GNA are perpendicular to field.

vector  $\vec{OF}_m$



### AT LOAD

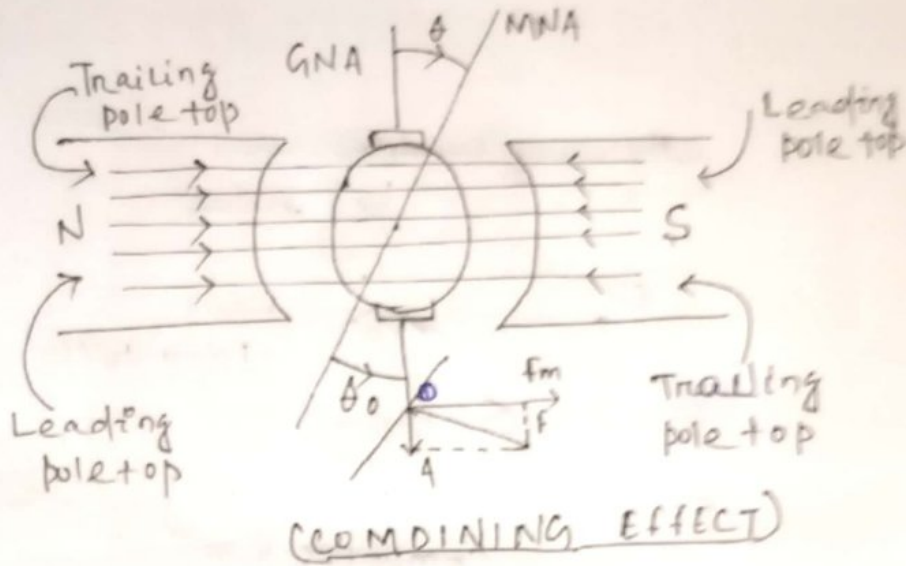
When the generator is loaded, it will produce a magnetic field considering only the armature current. The vector of  $OF_A$  represents both in magnitude and direction of the MMF due to armature winding.



COMBINING EFFECT

Under actual load condition, the above two effects exists simultaneously in the generator

The flux through the armature ( resultant flux ) is no longer uniform and symmetrical about the poles axis.

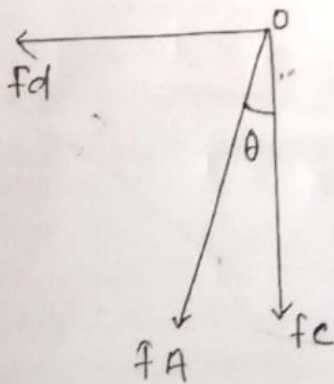


The resultant is  $OF$ , which is the vector sum of  $OF_m$  and  $OF_a$ . The new position of  $MNA$  is displaced from its original position by an angle  $\theta$ , because  $MNA$  is always perpendicular.

The armature MMF is found to lie in the direction of  $MNA$ .

The armature MMF is now represented by the of  $F_A$ . Which is vertical but is inclined by an angle  $\theta$  to the left. It can now be resolved in to two component.

We find that



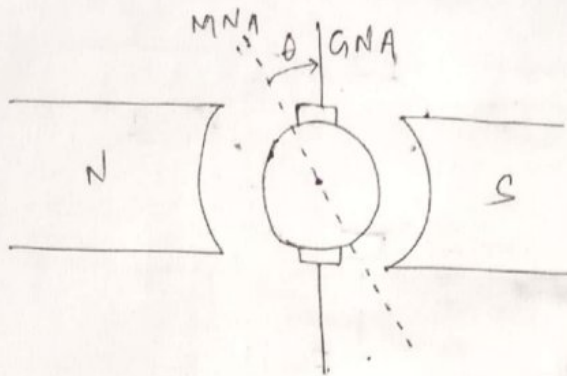


- I. Component  $O F_c$  is at right angle to the vector  $O F_m$  representing the main MMF . It produce distortion in main field and is hence called the cross magnetising or ~~distorting~~ <sup>distorting</sup> component of the armature reaction .
- II. The component  $O F_d$  is in direct opposition to  $O F_m$  which represent the main MMF . It exerts a demagnetizing influence on the main pole flux . It is the demagnetizing component of the armature reaction which weakens the main flux .

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### III. IN CASE OF MOTOR

When the machine act as motor . The current direction is reversed and hence MNA is shifted by an angle  $\alpha$  in the backward direction.



### DEMAGNETISING AMPERE TURN

Let  $Z$  = Total Number of armature conduction

$I$  = Armature current  $\rightarrow I_a/2 \rightarrow$  wave  
in each conductor  $\rightarrow I_a/p \rightarrow$  lap  
 $\phi_m$  = Mechanical degree in forward movment .

Total no. of armature conduction in angle

$$\angle AOC \text{ and } \angle BOD = \frac{Z}{360} \times 4\phi_m \quad \frac{Z}{360} \times 4\phi_m$$

No. of turn under  $\angle AOC$  and  $\angle BOD = \frac{Z}{360} \times 2\phi_m$  ( $\therefore$  two conduction constitute one turn)

$$\text{Demagnetizing ampere turns per pair of poles} = \frac{ZI}{360} \times 2\phi_m \quad \frac{2\phi_m}{360} \times ZI$$

$$\text{Demagnetizing ampere turns per pole} = \frac{ZI}{360} \times \phi_m \quad \frac{\phi_m}{360} \times ZI$$

## CROSS MAGNETISING AMPERE TURN

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$$\text{Total no of conductors per pole} = \frac{Z}{P}$$

$$\text{Demagnetizing conductors per pole} = \frac{Z}{360} \times 2\theta_m$$

$$Z \times \frac{2\theta_m}{360}$$

Cross magnetizing conductors pole = Total no of conductors per pole

$$= \frac{Z}{P} - \frac{Z}{360} \times \frac{2\theta_m}{360}$$

$$= Z \left( \frac{1}{P} - \frac{2\theta_m}{360} \right)$$

Cross magnetizing Ampere turns per pole

$$= \frac{Z}{P} - \frac{Z}{360} \times \frac{2\theta_m}{360}$$

$$Z I \left( \frac{1}{2P} - \frac{\theta_m}{360} \right)$$

$$= Z \left( \frac{1}{P} - \frac{2\theta_m}{360} \right)$$

$$\theta_{\text{mech}} = \frac{\theta_{\text{elect}}}{\text{Pair of Poles}} \quad (\text{If the angle is given in electrical degrees})$$

$$\theta_{\text{(mech.)}} = \frac{\theta_{\text{(elec.)}}}{\text{pair of poles}}$$

## COMMUTATION

The <sup>induced</sup> emf ~~include~~ in the armature conductors of a machine is an alternating in nature. The current in a conductor for in one direction when it is under north pole and in reverse direction when it is under south pole.

The reversal of current of current from (+) I to (-)I has to occur when two commutator segments to which the armature coil is connected are short circuited by a brush. This process is known as commutation period. The current in the coil has to reach its full value when in the reversed direction at the end of commutation period. If this does not happen the difference of current would pass from commutator to brush in the form of an A.C. arc. This arcing causes sparking pitting and roughing of the commutator surface.

Two major effect of disturb the commutation process are armature reaction and reactance voltage. The armature reaction causes a shift of the M.N.P (Magnetic Natural Plane) in the forward direction for the generator and in the back direction for the motors. For proper commutation in the commutator brush should short-circuited.

The commutator segments at the instants when voltage across them is zero. Because of the shift of M.N.P, the voltage between the segments has a finite value of the brush are in the G.N.P. The result is a current flowing between the short circuited commutator segments and arcing. The shifting of brushes the new M.N.P depends on the magnitude of load current Greater this current greater is the needed shifting

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greater is the load current since it is practically impossible to shift the brushes every time as the load current changes the brushes are always kept in G.N.P .

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In the end the commutation problem is due to the reactance voltage . The time of commutation is very short if a machine is running at 1000 R.P.M. and has 50 commutator segment then is segment moves under a brush and clears it again in time of 0.0012 and seconds . If the current changes from (+) 100A to (-)100A the rate of change of current is  $\frac{di}{dt} \cdot \frac{di}{dt} = 166667$  amp/ sec rate of changes of current is  $\frac{di}{dt}$  .

The coil under going commutation has an inductance therefore induced emf  $L \frac{di}{dt}$  is set up in the coil . Through the magnitude of inductance of very high and therefore the magnitude of induced emf coil be appreciable . This EMF is known as reactance voltage and oppose the reversal of current . Thus sparking occurs at the brushes .

Commutation problem can be minimized by different method .

- i. Emf commutation
- ii. By using interpoles
- iii. By resistance commutation
- iv. By using compensating winding

#### By Emf Commutation

In this method a voltage which cancels the reactance voltage is used to ensure good commutation . One way to cancel the reactance voltage is by shifting the brush a little farther than the M.N.P . so that they lie in the fringe of the field of the next pole . The Emf induced in the coil opposes the reactance voltage and opposes forces the reversal of current in the coil . However this method is not used because the extend of shifting of brushes depends on the load current and it is not practicable to shift the brushes every time as the load current changes .

#### By Using Interpoles Or Compoles

The interpoles helps on reducing the sparking due to commutation problem of current from A.C to D.C . They are small poles fixed to the yoke and placed in between to main poles . The windings of these poles has few turns of thick copper wire and is connected in series with the armature ckt . There fore the mmf of an interpole is proportional to armature current . The function of interpole is to .

- i. Ensure automatic neutrallization of reactance voltage .
- ii. Cancellation of cross-magnetization effect of armature reaction



## Reactance Commutation

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The next approach to achieve two commutation by the use of brushes with high contact resistance then the brushes made from other materials. Hence carbon is used as a brush universally. Also carbon has (-) ve temperature co-efficient of resistance properly.

## Compensating Winding

In order to neutralisation the cross magnetizing effect, compensating winding are used. It is used only in case of large machine. These windings are ambeded in slots in the pole shoes in series with armature in such a way that the current in them flows in opposite direction to that of in the armature induction directly below the poles-shoes.

$$\begin{aligned} \text{No. of compensating winding appear turns per pole} &= \frac{0.7 \times ZI}{2P} \\ &= \frac{0.7 \times Z}{2P} \end{aligned}$$

## DUMMY COILS

When a machine has a wave winding is very necessary to use extra coils to maintains the mechanical balance of the armature. This coil is completely insulated from the remaining winding and it is used for only mechanical balance.

It is known as dummy coils

$$Y_c = \frac{2(C \pm 1)}{P}$$

$$Y_c = \frac{Z \pm 2}{P}$$

C = No. of coils

$Y_c$  = Commutator pitch

P = No. of poles

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### EQUALLISER RING

The existence of many parallel path in a lap winding leads to be serious problems of circulating currents . The fluxes from all the poles are not exactly equal because of wear and tear on the bearing, the air gap doesnot remain uniform around the whole periphery . As the armature conductor rotates the voltages induce in different parallel paths will be slightly different . Hence armature winding . Due to circulating current energy loss and heating effect results . Therefore equalizer rings are necessary which will be connected to make some potential different parallel path . Each rings are insulated from which others . By using equalizer rings induced emf can be made equal no. of equalizer rings

$$\begin{aligned} &= \frac{\text{Total No. of Conductors}}{\text{N. of Pair of Poles}} \\ &= \frac{Z}{\frac{P}{2}} \\ &= \frac{2Z}{P} \quad (\because \text{No. of Equalliser Rings} = \frac{2Z}{P}). \end{aligned}$$

### CRITICAL FIELD RESISTANCE OF A SHUNT GENERATOR

The maximum emf generated is  $o_c$ , if the shunt field resistance is increased, then the maximum generated emf is represented by  $o_c$ . so that if becomes a tangent to the curve . the value of field resistance corresponding to the point of intersection of the field resistance for a given speed again it is seen that ,if the field resistance is increased further beyond the critical resistance the generator does not excite at all in other words the critical field resistance  $R_c$  of a shunt generator is the maximum value of field resistance beyond which the generator can't build of voltage .

### CRITICAL SPEED OF A SHUNT GENERATOR

The speed for which a given shunt field resistance acts as critical field resistance is known as critical speed .

If  $E_1$  and  $E_2$  be the respective values induced emf for the same excitation current at speed  $N_1$  and  $N_2$

$$\text{Then } \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

### CHARACTERISTICS OF D.C GENERATOR

There are three different types of characteristic

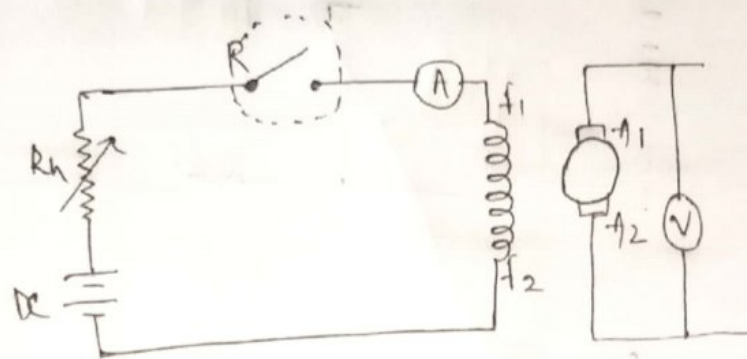
- (1)-NO-load /magnetization/ open circuit characteristics (o.c.c) ✓
- (2)-Internal characteristics ✓
- (3)-Load/External characteristics ✓

(1)-NO LOAD/MAGNETISATION/OPEN CIRCUIT CHARACTERISTICS(O.C.C)

It is graphical relationship between generated emf and field current ( $E_g \sim I_f$ )

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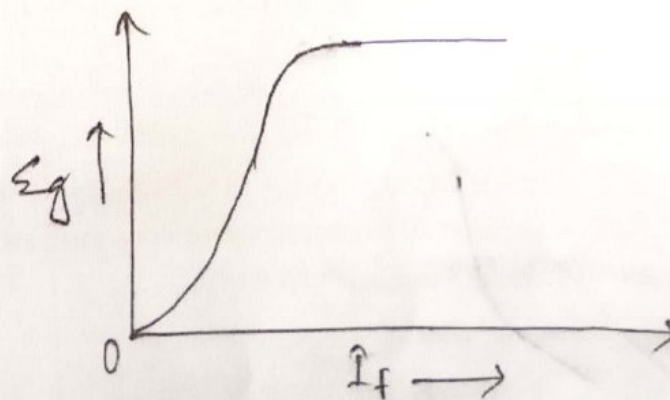
FOR A SEPARATELY EXCITED D.C GENERATOR



For a separately excited DC generator

Let the switch is open, but the generator is driven by some external source (Prime-mover or dc motor). It is seen that the generated emf is zero, since the flux is zero.

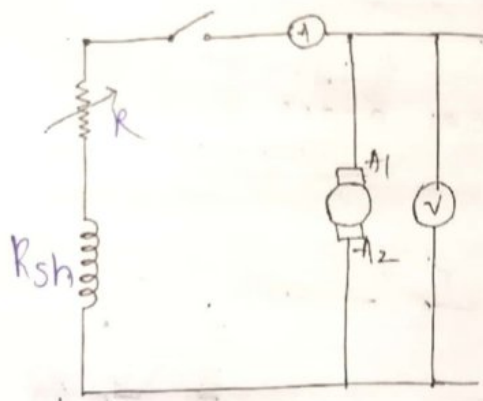
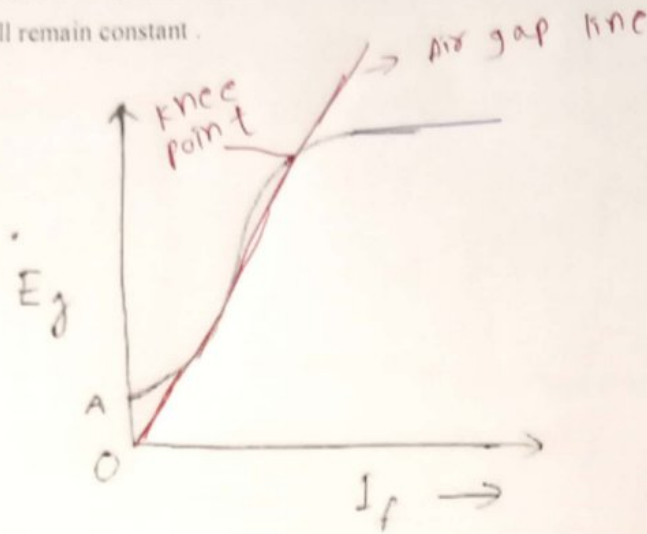
Now the switch is closed and the field current increases gradually. It is seen that as the field current increases, the generated emf is proportional to the field flux. This will continue till saturation. After saturation of magnet field, the field current may be increases but the field flux remains constant. So the generated emf remains constant even if the field current increases.





# FOR SELF-EXCITED DC GENERATOR

When the field current is zero, the EMF induced in the armature is 'OA'. This is due to the presence of residual magnetization. Again if the field current increases the EMF increase and it will continue up to the point of saturation. After saturation the field current may increase but field flux remain constant. So that EMF induced will remain constant.



## (2). INTERNAL CHARACTERISTICS:-

It is the graphical relationship between voltage and armature current  $E_0 \approx I_a$ . when the armature current is zero, the generated EMF is equal to the no load voltage. As the armature current increase the armature resistance drop is  $(I_a R_a \text{ drop})$  increases. So the terminal voltage decreases. At heavy loads, due to armature reaction the terminal voltage decreases to a lower value.

We know that,

$$E_g = V + I_a R_a$$

$$V = E_g - I_a R_a$$

### (3) EXTERNAL/LOAD CHARACTERISTICS :- (V ~ I)

It is the graphical relationship between the two terminal voltage and the load current .

V~I

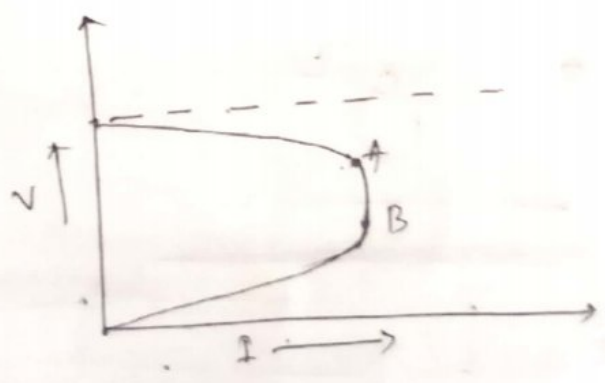
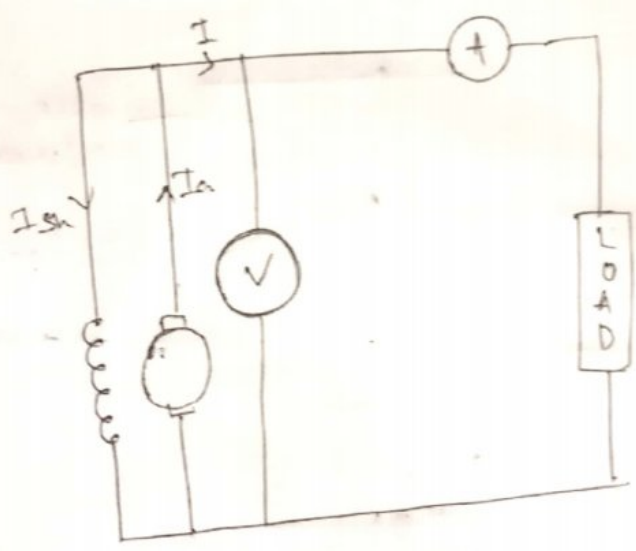
### FOR SHUNT GENERATOR

It is seen that when the load current increase, the terminal voltage decreases. As the load current increase  $I_a R_a$  drop increases .

But at point 'A' if further the load increases, the terminal voltage decreases suddenly.

This is due to the armature reaction .

$$V = E_g - I_a R_a$$



The drops are due to,

- (1) Armature resistance drop ( $I_a R_a$  drop)
- (2) Armature reaction
- (3) The combining effect, the terminal voltage decreases suddenly as the load current increase, it is represented by A to B.

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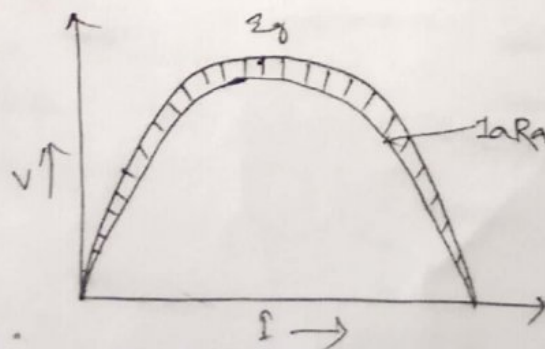
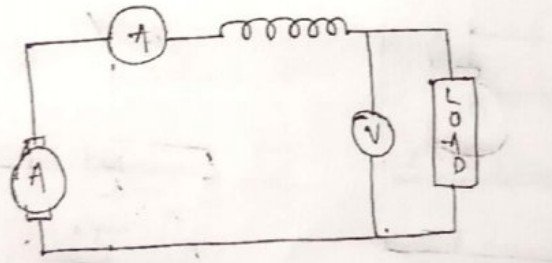
If further the load increases, the generator will come to its instability condition, which is shown by dotted lines. If the load increases further the terminal voltage decreases to a very lower value and the generator cannot maintain its stability. Automatically it will come to 'OFF' position.

This is known as dropping characteristic of D.C shunt generator. Due to this reason it is suitable for lighting purpose and battery charging purpose.

### FOR SERIES GENERATOR:-

It is seen that load current increases the terminal voltage increases. This is due to the load current passes through the field..It continue up to the point of saturation. After saturation, if the load current increases, then the terminal voltage decreases. This is known as rising characteristics of a D.C series generator so it is used as a booster.

$$V = E_g - I_a R_a - I_a R_{se}$$





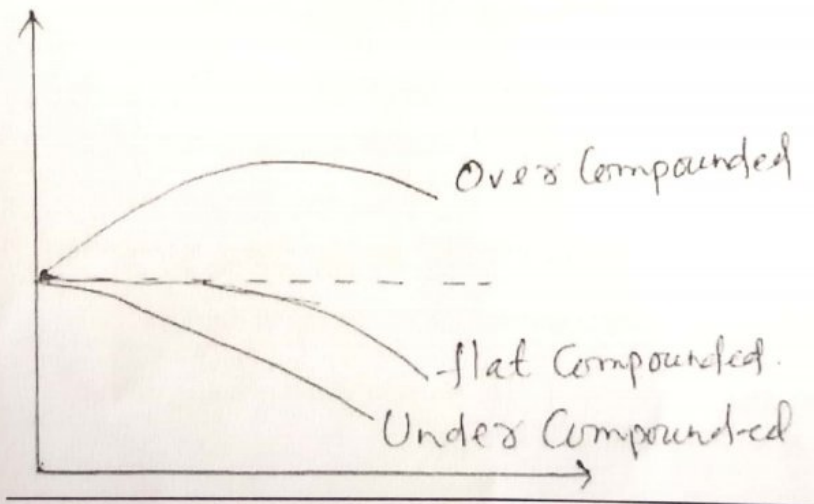
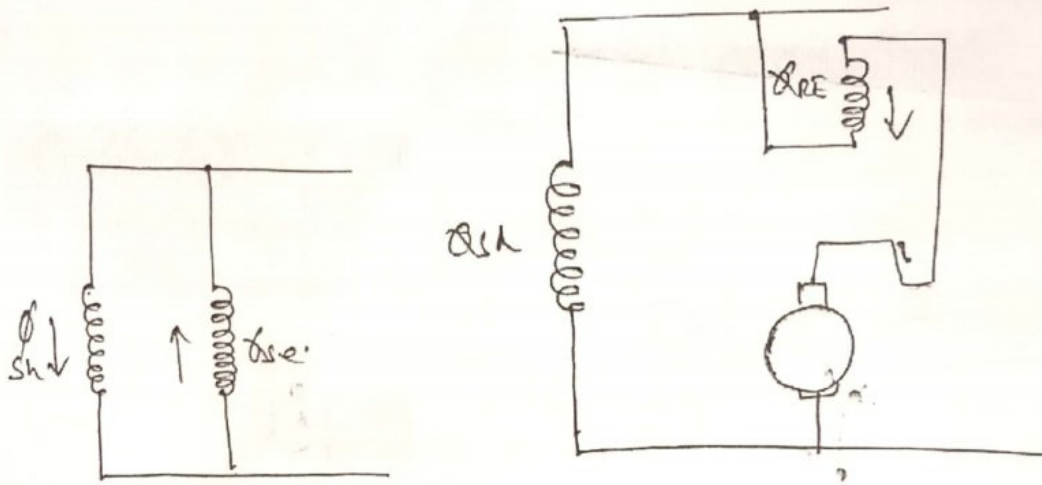
Curve.....

The drops are due to

- (1)  $I_a R_a$  drop
- (2)  $I_a R_{se}$  drop
- (3) Armature Reaction

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### FOR COMPOUND GENERATOR:-



Differential Compound D.C generator Net Phase =  $\phi_{sh} - \phi_{sc}$

Commutatively Compound D.C Generator Net Phase =  $\phi_{sh} + \phi_{sc}$

## OVER COMPOUND/COMMULATIVELY

### COMPOUND:-

In case of commulatively compounded D.C Generator the series field flux aids to the shunt field flux. As the load current increases series field flux increases. As the load current increases, the terminal voltage increases. If the terminal voltage is more than the no load voltage then it is known as over compounded D.C Generator.

## UNDER COMPOUNDED/DIFFERENTILLY

### COMPOUNDED:-

In case of defferentially compounded D.C Generator the series field flux opposes the shunt flux. As the load current increases, the net flux decreases. Hence generated EMF decreases. As the load current increases the terminal voltage decreases, then no load voltage as the load current increases. It is known as external characteristics of under compounded D.C Generator.

### FLAT COMPOUNDED:-

The change in no load voltage to full load voltage is negligible is known as flat compounded D.C Generator. As the load current increases, the terminal voltage decreases slightly.

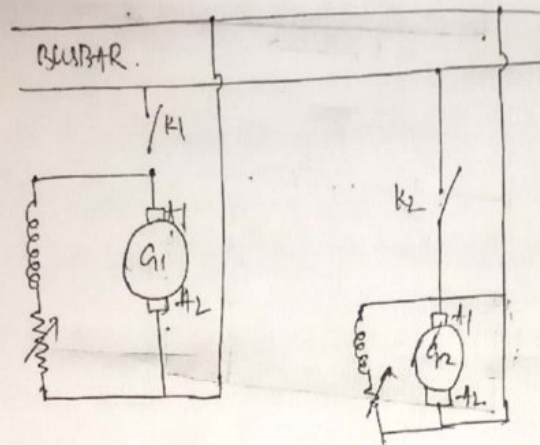
## CONDUCTION FOR BUILT UP A SELF-EXCITED

### D.C GENERATOR:-

- \*There must be some residual magnetism in the poles.
- \*For the given direction of rotation, the shunt field poles should be connected properly to the armature.
- \*If excited on open circuit, its shunt field resistance should be less than the critical resistance.
- \*If excited on load, then the shunt field resistance should be more than a certain minimum value of resistance which is given by internal characteristics.
- \*The series generator should be started with load.

# PARALLEL OPERATION OF A DC GENERATOR

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## CONDITION FOR PARALLEL OPERATION

- I. Polarity must be maintained.
- II. The terminal voltage of generator must be equal to the bus bar voltage  $V_1 = V_2 = V$
- III. The load sharing should be equal.

### LOAD EQUAL

Load sharing two generators which have unequal no-load voltage.

Let,  $E_1$  = No load voltage of  $G_1$

$E_2$  = No load voltage of  $G_2$  [?????]

$R_{a1}$  = Armature resistance of  $G_1$

$R_{a2}$  = Armature resistance of  $G_2$

$V$  = Bus bar voltage

= Common terminal voltage

$$\Rightarrow I_{a1} = \frac{E_1 - V}{R_{a1}}$$

$$\Rightarrow I_{a2} = \frac{E_2 - V}{R_{a2}}$$

$$\Rightarrow \frac{I_{a2}}{I_{a1}} = \frac{E_2 - V}{E_1 - V} \cdot \frac{R_{a1}}{R_{a2}}$$

?



$$\Rightarrow \frac{K_2 N_2 \phi_2 - V}{K_1 N_1 \phi_1 - V} = \frac{R_{a1}}{R_{a2}}$$

From the above equation it is seen that the bus bar voltage can be kept constant by increasing  $\phi_2$  or  $N_2$  or by reducing  $N_1$  and  $\phi_1$ .

$N_1$  and  $N_2$  are changed by the help of resulting shunt field resistance .

- Two parallel shunt Generator having equal no-load voltage share the load in a ratio that the load current of each machine produces the same drop in each generator .
- In case of 2 generator having un-equal no-load voltage ,the load current produces sufficient voltage drop in each .so as kept their terminal voltage same .

## SERIES GENERATOR IN PARALLEL

Suppose  $E_1$  and  $E_2$  are initially equal generators, supply equal current and have equal series resistance .suppose  $E_1$  increases slightly . so that  $\frac{E_1}{E_2}$

In this case  $I_1$  becomes grater then  $I_2$  .Now the field of machine  $G_1$  is strong then .Then increases  $E_1$  further the field of machine  $G_1$  is weakened .

Then decreases  $E_2$  further a final stage is reached , when the machine  $G_1$  is supplied not only the whole load but also power to the machine  $G_2$  which starts remaining as a motor .This can be prevalent by using equal bus bar machines pass approximately equal currents to the load.

It essential that series field resistances are inversely proportional to the generator rating .

## COMPOUND GENERATOR IN PARALLEL

It is same as in series Generator for maintaining division of load from no- load to full-load .

- I. The regulation of each generator is same
- II. The series field resistance are inversely proportional to the generator rating .

## Generator

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1. A short shunt compound DC generator delivers a load current of 30 A at 220 V, and has an armature, series field and shunt field resistance of  $0.05 \Omega$ ,  $0.30 \Omega$  and  $200 \Omega$  respectively. Calculate the induced emf ~~current~~ and armature current. Allow 1.0 V per brush for contact drop.

Sol<sup>n</sup>

Given data:

$$I = 30 \text{ A}$$

$$V = 220 \text{ V}$$

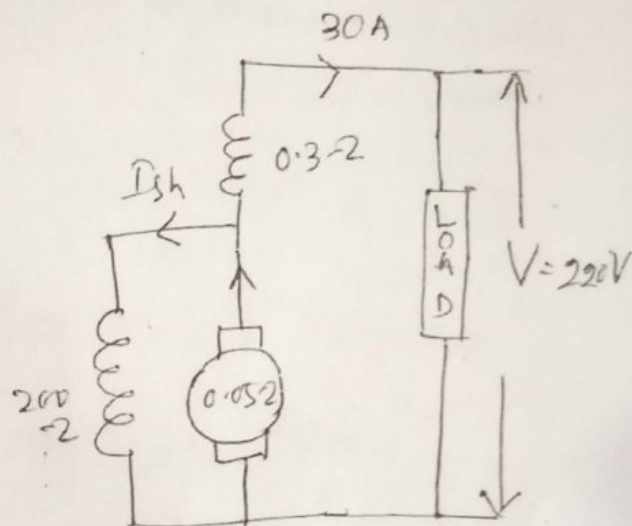
$$r_a = 0.05 \Omega$$

$$R_{se} = 0.30 \Omega$$

$$R_{sh} = 200 \Omega$$

$$I_a = ?$$

$$E_g = ?$$



Shunt field Voltage drop =

$V + \text{series field drop}$

$$= 220 + I \times 0.03$$

$$= 220 + 30 \times 0.03 = 229 \text{ V}$$

$$I_{sh} = \frac{220}{200} = \frac{229}{200} = 1.145 \text{ A}$$

$$I_a = I + I_{sh} = 30 + 1.145 = 31.145 \text{ A}$$

$$E_g = V + I R_{se} + I_a r_a$$

$$= 220 + 30 \times 0.03 + 31.145 \times 0.05$$



2. A 4-pole, d.c. shunt generator with a shunt-field resistance of  $100\ \Omega$  and an armature resistance of  $1\ \Omega$  has 378 wave-connected conductors in its armature. The flux per pole is  $0.02\ \text{wb}$ . If a load resistance of  $10\ \Omega$  is connected across the armature terminals and the generator is driven at  $1000\ \text{rpm}$ , calculate the power absorbed by the load.

27.

Sol<sup>n</sup>

Given data

$$P = 4$$

$$R_{sh} = 100\ \Omega$$

$$r_a = 1\ \Omega$$

$$Z = 378$$

$$A = 2$$

$$\phi = 0.02\ \text{wb}$$

$$R_L = 10\ \Omega$$

$$N = 1000\ \text{rpm}$$

$$E_g = \frac{P\phi ZN}{60A} = \frac{4 \times 0.02 \times 378 \times 1000}{60 \times 2}$$

$$= 252\ \text{V}$$

$V$  is the terminal voltage.

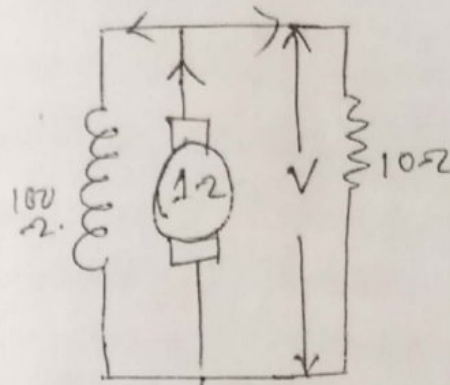
$$I_a = \frac{V}{10}, \quad I_{sh} = \frac{V}{100}\ \text{A}$$

$$\text{Armature Current} = \frac{V}{10} + \frac{V}{100}$$

$$= \frac{11V}{100}$$

$$V = E_g - \text{armature drop}$$

$$= 252 - I_a R_a = 252 - \frac{11V}{100} \times 1 = 252 - 0.11V$$



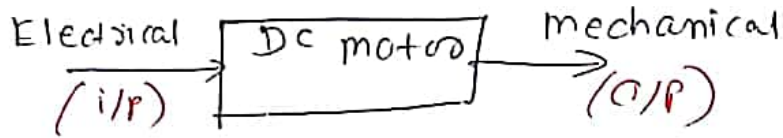


# DC Motor

(1)

→ ~~DC motor is a machine that converts i/p DC signal supply to~~

→ DC motor is a machine that converts ~~mechanical~~ Electrical energy to mechanical energy.



→ It's operation is based on the Lorentz's principle.

i.e. Whenever a current carrying conductor is placed inside a magnetic field, the conductor experiences a mechanical force.

→ The direction of the force is given by FLHR & magnitude is  $F = BIL$  Newton

→ Generator action will also take place in the motor, due to motor action, armature conductors are rotating in the magnetic field, they cut the magnetic flux & dynamically induced emf is produced.

The direction of this induced emf is acting opposite to the supply voltage. That's why this induced emf is called back emf or counter emf.

→ The armature current is proportional to the potential difference  $(V - E_b)$ .

$$I_a = \frac{V - E_b}{R_a}$$

where  $E_b$  = back emf

$R_a$  = Armature resistance

$V$  = supply voltage

significance of back emf

→ The presence of back emf makes the DC motor a self-regulating machine i.e. it makes the motor to draw as much armature current as is just sufficient to develop the torque required by the load

$$I_a = \frac{V - E_b}{R_a}$$

→ when the motor is running on no-load, small torque is required to overcome the friction & windage losses, so  $I_a$  is small & the back emf is nearly equal to the applied voltage.

→ If the motor is suddenly loaded, the first effect is to cause the armature to slow down. Therefore, the speed at which the armature conductors move through the field is reduced & hence the back emf  $E_b$  falls. The decreased back emf allows a larger current to flow through the armature & large current means increased driving torque.

→ If the load on the motor is decreased, the driving torque is momentarily in excess of the requirement so that armature is accelerated.

Voltage eqn of DC motor

$V$  = Applied voltage

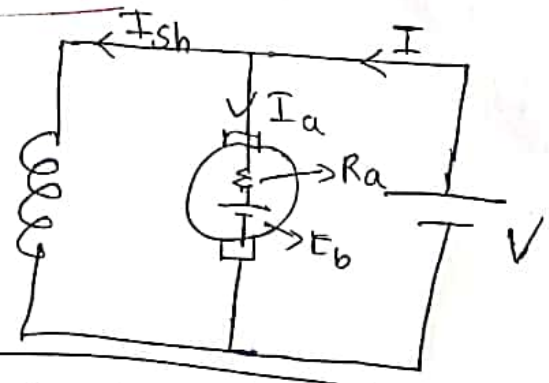
$E_b$  = back emf

$R_a$  = Armature resistance

$I_a$  = Armature current

$$I_a = \frac{V - E_b}{R_a}$$

$$\Rightarrow V = E_b + I_a R_a$$



Power equation

$V = E_b + I_a R_a$   
multiply  $I_a$  on both sides

$V I_a = E_b I_a + I_a^2 R_a$

$V I_a$  = Electrical power supplied to armature  
 $E_b I_a$  = Electrical power developed by armature.  
 $I_a^2 R_a$  = Armature cu. loss

$P_{in} = P_{mech} + \text{Arm. cu loss}$

$P_{mech} = E_b I_a = T_{cm} \times \omega$   
 $T_{cm} = \frac{P_{mech}}{\omega} = \frac{E_b I_a}{\omega}$

Cond<sup>n</sup> for maximum power

Mechanical power developed by the motor is  $P_m = E_b I_a$

Now  $P_m = V I_a - I_a^2 R_a$

As ( $V, R_a$  fixed)

$\frac{dP_m}{dI_a} = V - 2 I_a R_a = 0 \quad I_a R_a = \frac{V}{2}$

Now  $V = E_b + I_a R_a = E_b + \frac{V}{2} \quad \therefore E_b = \frac{V}{2}$

$\therefore$  Hence mechanical power developed by the motor is maximum when back emf is equal to the applied voltage.

Armature torque of Dc motor

$\rightarrow$  Torque is the turning moment of a force about an axis & is measured by the product of Force ( $F$ ) & radius ( $r$ ) at right angle to which the force acts





$$W = F \times r \times (n \cdot 2\pi)$$

Work done by this force in one revolution

$$= \text{force} \times \text{distance}$$

$$= F \times 2\pi r \text{ Joule}$$

$$\text{Power developed} = (F \times 2\pi r) n \text{ Joule/second or watt}$$

$$= (F \times r) \times 2\pi n \text{ watt}$$

$2\pi n$  = Angular velocity  $\omega$  in radians/second

$$\text{and Torque (T)} = F \times r$$

$$\text{Power developed} = T \times \omega \text{ watt or } \boxed{P = \omega T \text{ watt}}$$

Moreover, if  $n$  is rpm,

$$\omega = \frac{2\pi n}{60} \text{ rad/sec}$$

$$P = \omega T = \frac{2\pi n}{60} \times T \quad P = \frac{2\pi}{60} \cdot nT$$

$$\boxed{P = \frac{nT}{2.55}}$$

### Armature torque of a motor

Let  $T_a$  be the torque developed by the armature of a motor running at  $n$  r.p.s.

$$P = T_a \times 2\pi n \text{ watt}$$

We know that elect. power ~~developed~~ converted into mechanical power  $P_m = E_b I_a \text{ watt}$ .

$$P = P_m$$

$$T_a \times 2\pi n = E_b I_a$$

$$T_a \times 2\pi n = \frac{P \phi Z n}{60 A} \times I_a$$

$$T_a = \frac{1}{2\pi} \frac{P \phi Z}{60 A} \times I_a$$

$$\boxed{T_a = 0.159 \phi Z I_a \left(\frac{P}{A}\right) \text{ N-m}}$$

$$\boxed{T_a \propto \phi I_a}$$

For series motor

$$\phi \propto I_a \quad \text{so} \quad T_a \propto I_a^2$$

For shunt motor ( $\phi = \text{const}$ )

$$T \propto \frac{E_b I_a}{2\pi N} \quad \text{so} \quad T_a \propto I_a$$

$$T_a = \frac{E_b I_a}{2\pi N} \text{ in } \delta \text{ P S}$$

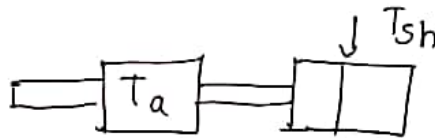
$$\text{in } \delta \text{ P M, } T_a = \frac{E_b I_a}{(2\pi N/60)}$$

$$= 60 \frac{E_b I_a}{2\pi N} = \frac{60}{2\pi} \cdot \frac{E_b I_a}{N}$$

$$T_a = 9.55 \frac{E_b I_a}{N} \text{ N-m}$$

Shaft torque ( $T_{sh}$ )

→ The torque which is available for doing useful work is known as shaft torque ( $T_{sh}$ ).



$$\text{O/P} = T_{sh} \times 2\pi N \text{ watt} \quad \begin{matrix} N \text{ in } \delta \text{ P S} \\ T_{sh} \text{ in N-m} \\ N \text{ in } \delta \text{ P M} \end{matrix}$$

$$T_{sh} = \frac{\text{O/P in watts}}{2\pi N}$$

$$T_{sh} = \frac{60}{2\pi} \frac{\text{O/P in } \delta \text{ P M}}{N} \Rightarrow 9.55 \frac{\text{O/P}}{N} \text{ N-m}$$

$$T_{sh} = 9.55 \times \frac{\text{O/P}}{N} \text{ N-m}$$

→  $T_{sh} < T_a$ . because of iron & frictional loss

$$T_a - T_{sh} = \text{lost torque}$$

## Speed of DC motor

$$E_b = V - I_a R_a$$

$$\Rightarrow \frac{P \phi Z N}{60 A} = V - I_a R_a \Rightarrow N = \frac{V - I_a R_a}{\phi} \times \frac{60 A}{P Z}$$

$$\Rightarrow N = K \left( \frac{V - I_a R_a}{\phi} \right) \Rightarrow N = K \frac{E_b}{\phi}$$

$$\therefore N \propto \frac{E_b}{\phi}$$

→ if DC motor has initial values  $N_1, \phi_1, E_{b1}$   
& final values  $N_2, \phi_2, E_{b2}$

$$\text{then } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

For shunt motor

$$\phi_1 = \phi_2 \quad \therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

For series motor

$$\phi \propto I_a \quad (\text{before saturation}) \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

## Speed regulation

The speed regulation of a motor is the change in speed from F.L. to No-load and is expressed as a percentage of speed at F.L.

$$\% \text{ Speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100$$

## Torque & Speed of a DC motor

$$i) N \propto \frac{E_b}{\phi}$$

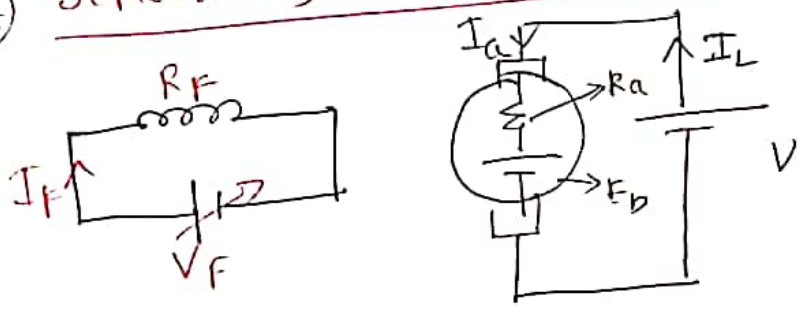
$$ii) T \propto \phi I_a$$



→ If flux is decreased slightly by decreasing field current.

- ①  $E_b$  drops instantly (but speed remains constant due to inertia of heavy armature)
- ② As  $E_b \downarrow$ ,  $I_a \uparrow$ ,  $I_a \uparrow = \frac{V - E_b}{R_a}$ . A small deduction in flux produces a proportionately large current in armature.
- ③  $T_a \propto \phi I_a \rightarrow \uparrow$  (more). A small decrease in flux is more than counterbalanced by a large increase in  $I_a$  with a result that net increase in  $T_a$ .

⊛ Separately excited DC motor

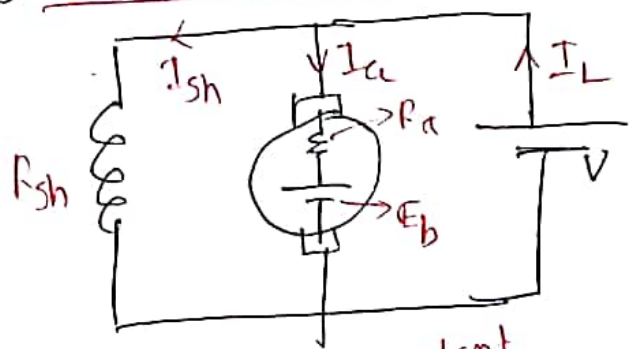


$$E_b = V - I_a R_a - B.D$$

$$I_L = I_a$$

∴ if B.D neglected  
 $P_{input} = V I_L$

⊛ DC shunt motor



$$I_L = I_a + I_{sh}$$

$$I_a = I_L - I_{sh}$$

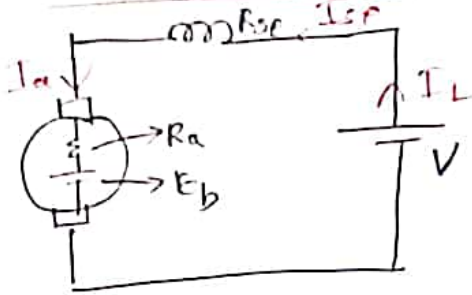
$$I_{sh} = \frac{V}{R_{sh}}$$

$$E_b = V - I_a R_a - B.D.$$

$\phi \propto I_{sh}$ ,  $I_{sh} = \frac{V}{R_{sh}}$   
 ∴ Flux constant.

→ DC shunt motor is a constant flux motor.

4) DC series motor



$$I_L = I_{se} = I_a$$

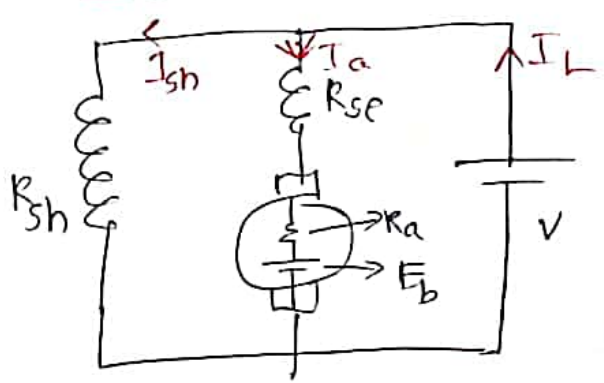
$$V = E_b + I_a R_a + I_a R_{se} \quad \text{--- B.D}$$

$$E_b = V - I_a R_a - I_a R_{se} \quad \text{--- B.D}$$

$$E_b = V - I_a (R_a + R_{se}) \quad \text{--- B.D}$$

5) DC compound motor

→ long shunt



$$I_L = I_a + I_{sh}$$

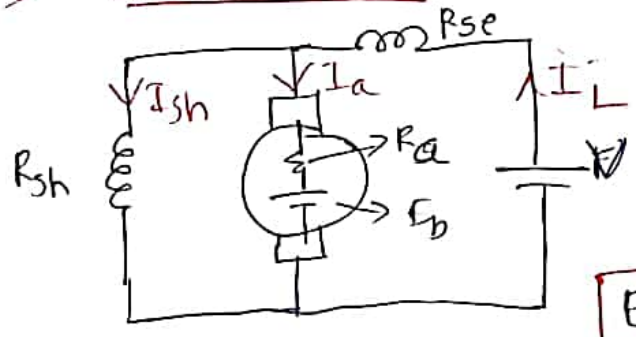
$$I_a = I_L - I_{sh}$$

$$\phi_{se} \propto I_a$$

$$\phi_{sh} \propto I_{sh} = \frac{V}{R_{sh}}$$

$$E_b = V - I_a (R_a + R_{se}) \quad \text{--- B.D}$$

→ short shunt



$$\phi_{se} \propto I_L$$

$$\phi_{sh} \propto I_{sh}$$

ckc  $I_L = I_a + I_{sh}$

$$I_a = I_L - I_{sh}$$

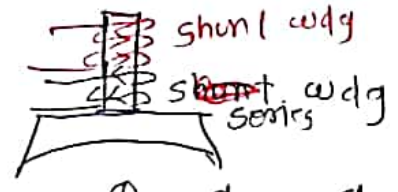
$$E_b = V - I_a R_a - I_L R_{se} \quad \text{--- B.D}$$

$$I_{sh} = \frac{V - I_L R_{se}}{R_{sh}}$$



$$\phi_{se} \uparrow \uparrow \phi_{sh} \quad \phi_T = \phi_{se} + \phi_{sh}$$

→ cumulative compound motor



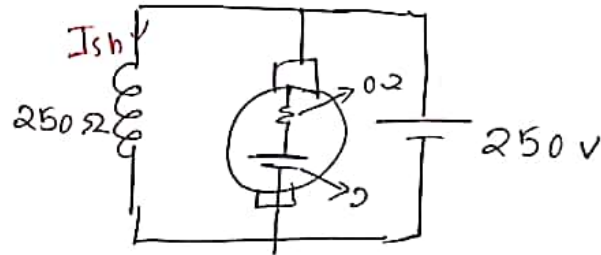
$$\phi_T = \phi_{sh} - \phi_{se}$$

→ differential compound motor

Q1) (1) A 250V shunt motor runs at 1000 r.p.m at No-load and takes 8A. The total armature & shunt field resistances are  $0.2\Omega$  &  $250\Omega$  respectively. Calculate the speed when loaded & taking 50A?

A:  $\frac{N}{N_0} = \frac{E_b}{E_{b0}} \times \frac{\phi_0}{\phi}$  since shunt motor flux is constant,  
 $\phi = \phi_0$

SO.  $\frac{N}{N_0} = \frac{E_b}{E_{b0}}$



$I_{sh} = \frac{250}{250} = 1A$

$E_{b0} = V - I_{a0} R_a$   
 $= 250 - 7 \times 0.2$   
 $= 248.6 \text{ Volt}$

$I_{a0} = 8 - 1 = 7$  at No-load

$E_b = V - I_a R_a$   
 $= 250 - 49 \times 0.2$   
 $= 240.2 \text{ Volt}$

$I_a = 50 - 1 = 49$  at load

$\frac{N}{1000} = \frac{240.2}{248.6} \Rightarrow N = \frac{966.21}{966.21} \times 1000 \text{ rpm}$

Q2) A DC series motor operates at 800 r.p.m with a line current of 100A from 230 V mains.  $R_a = 0.15\Omega$ ,  $R_{se} = 0.1\Omega$ . Find the speed at which motor runs at a line current of 25 A. Assuming flux at this current is 95% of the flux at 100A?

A:  $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$   $\phi_2 = 0.95\phi_1$   $\frac{\phi_1}{\phi_2} = \frac{1}{0.95}$

$E_{b1} = 230 - (0.15 + 0.1) \times 100 = 205 \text{ Volt}$

$E_{b2} = 230 - 25 \times 0.25 = 223.75 \text{ Volt}$

$\frac{N_2}{800} = \frac{223.75}{205} \times \frac{1}{0.95} \Rightarrow N_2 = 1940 \text{ r.p.m}$



Q) A 4 pole, 220V-shunt motor has 540 lap wound conductors. It takes 32 AMP from the mains supply & develops o/p power of 5.595 kW. The field winding takes 1A.  $R_a = 0.09 \Omega$ ,  $\phi = 30 \text{ mwb}$   
 Calculate i) speed ii) torque in N-m

$A \Rightarrow Z = 540$

LAP,  $A = P = 4$

$I_L = 32 \text{ AMP}$ ,  $\phi = 30 \times 10^{-3}$

$I_{sh} = 1 \text{ AMP}$

$I_a = 32 - 1 = 31 \text{ AMP}$

$E_b = V - I_a R_a$

$= 220 - 31 \times 0.09 = 217.2 \text{ volt}$

$E_b = \frac{P \phi Z N}{60 A} \Rightarrow N = \frac{E_b \times 60 \times A}{P \phi Z} = \frac{217.2 \times 60}{30 \times 10^{-3} \times 540}$

$N = 804.1 \text{ rpm}$

$\rightarrow T_{sh} = 9.55 \times \frac{\text{o/p in watts}}{N}$   
 $= 9.55 \times \frac{5.595 \times 10^3}{804.1} = 66.5 \text{ N-m}$

Formula in motor

$E_b = \frac{P \phi Z N}{60 A}$        $E_b = V - I_a R_a$  (shunt)

$T_a = 0.159 \phi Z I_a \left(\frac{P}{A}\right) \text{ N-m}$

$T_a = 9.55 \times \frac{E_b I_a}{N}$

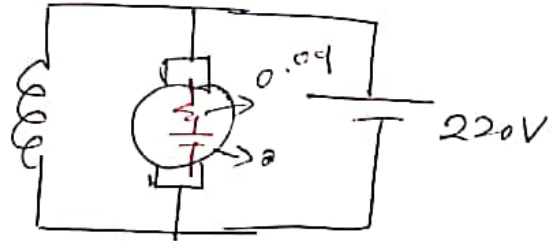
$T_{sh} = 9.55 \times \frac{\text{o/p}}{N}$

$T_a - T_{sh} = \text{lost torque}$

$T \propto \phi I_a$

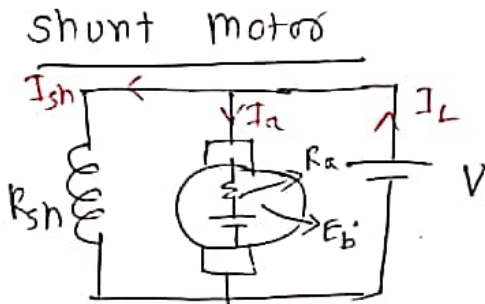
$T \propto I_a \rightarrow \text{shunt } (\phi = \text{const.})$

$T \propto I_a^2 \rightarrow \text{series}$



characteristics of DC motor

- ① Speed vs Armature current ( $N$  vs  $I_a$ )
  - ② Torque vs Armature current ( $T$  vs  $I_a$ )
  - ③ Speed vs torque ( $N$  vs  $T$ )
- Electrical characteristics  
→ mechanical characteristics



$\phi \propto I_{sh} = \frac{V}{R_{sh}} = \text{constant}$

$N \propto \frac{E_b}{\phi} \propto \frac{V - I_a R_a}{\phi}$

①  $N$  vs  $I_a$

At No. load

$I_a = 0, N \propto V$

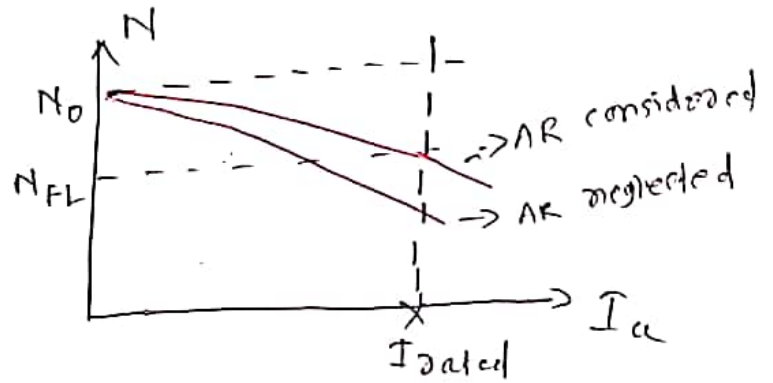
→ when load ↑,  $I_a$  ↑

$N \propto V - (I_a R_a) ↑$

due to Armature reaction

$\phi ↓, N ↑, N \propto \frac{1}{\phi ↓}$

→ DC shunt motor is a constant speed motor.

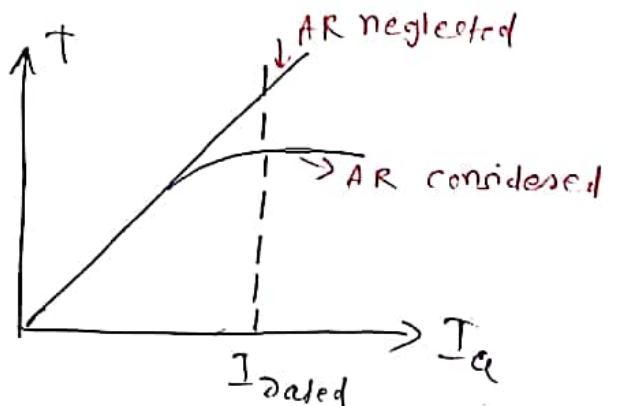


②  $T$  vs  $I_a$  ∴  $T \propto \phi I_a$

$T \propto I_a$  as  $\phi = \text{constant}$ .

if AR considered

$T \propto \phi I_a$



∴  $T$  vs  $I_a$  characteristics is linear.

③ N vs T

$T \propto I_a \therefore T = k_t I_a \quad I_a = \frac{T}{k_t}$

$N \propto \frac{V - \frac{T}{k_t} R_a}{\phi}$

At N.L  
 $I_a = 0, T_{em} = 0, N \propto \frac{V}{\phi}$

loaded  
load  $\uparrow, T_{em} \uparrow, N \downarrow$

with AR  
 $\phi \downarrow, N \uparrow,$

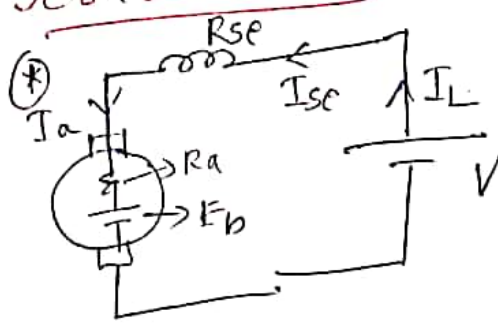
Application

Fan, centrifugal pump, lathe, machine tools,  
line shunting

Important

The direction of rotation of D.C. motor can be reversed either by changing the field terminals or armature terminals but not both.

Series motor



⊗  $\phi \propto I_a$  (before saturation)

$N \propto \frac{E_b}{\phi} \propto \frac{V - I_a(R_a + R_{se})}{I_a}$

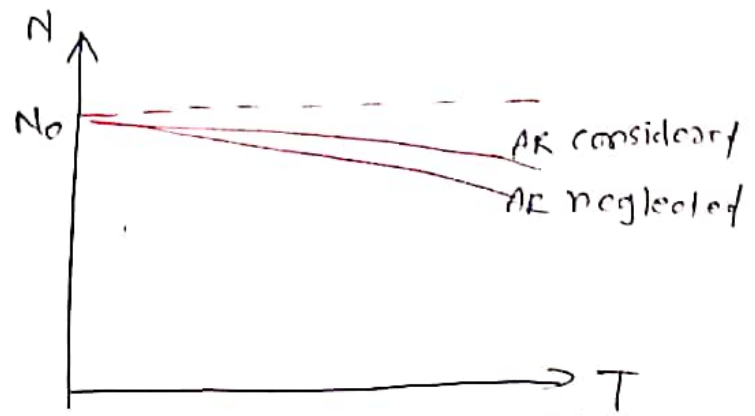
$N \propto \frac{1}{I_a}$

$T \propto \phi I_a$   
 $T \propto I_a^2$

After saturation

$\phi = \phi_{sat} = \text{constant}$   
 $N \propto V - I_a(R_a + R_{se})$

$T \propto I_a$



(Speed falls somewhat as the load torque increases)



① N vs I<sub>a</sub>

At No-load

$I_a \approx 0, (N \propto \frac{1}{I_a \approx 0})$

$E_b = V - I_a (R_a + R_{se})$

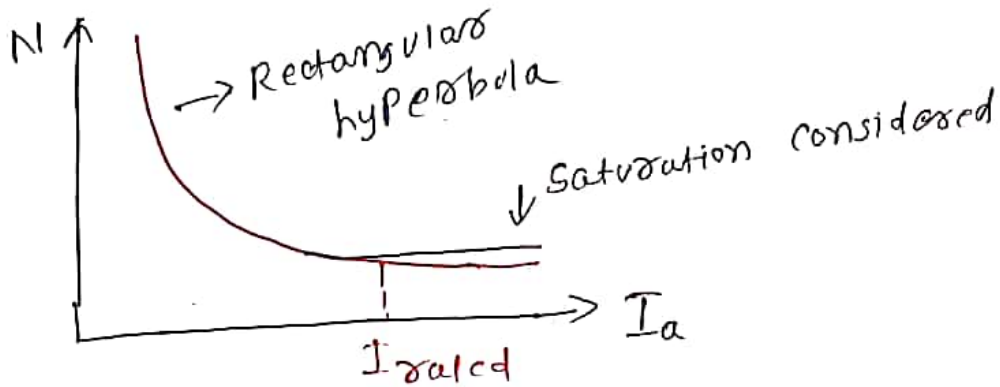
$I_a (R_a + R_{se}) \rightarrow$  drop very small

→ No-load speed of series motor is dangerously high speed.

→ Never start DC series motor without load.

At load

Mech load ↑, I<sub>a</sub> ↑, N ↓



Note =

Series motor is a variable flux machine. At overload cond<sup>n</sup> field poles saturated and flux becomes constant.

② T vs I<sub>a</sub>

$T \propto \phi I_a$

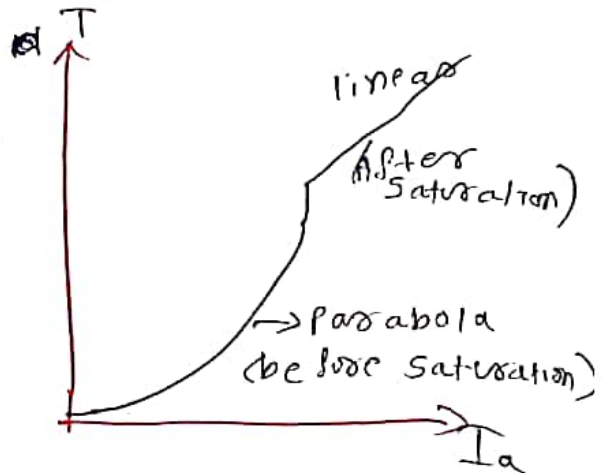
→ upto magnetic saturation

$T \propto I_a^2 \rightarrow$  parabola

After saturation  $\phi = \text{constant}$

$T \propto I_a \rightarrow$  straight line

$\phi \propto I_a$



→ Series motor has high starting torque.

③ N vs T

$$I_a \propto \sqrt{T}$$

$$N \propto \frac{1}{I_a} \quad \text{i.e.} \quad \boxed{N \propto \frac{1}{\sqrt{T}}}$$

No-load

$$T_{em} = 0, N = \infty$$

load  $\uparrow$ ,  $T_{em} \uparrow$ ,  $N \downarrow$

i.f.  $P = \text{const.}$   $P = \omega T$

$$\boxed{N \propto \frac{1}{T}}$$

→ Dc series motor behaves as a constant power device.

Applications

→ It is a variable speed motor & it has high starting torque.

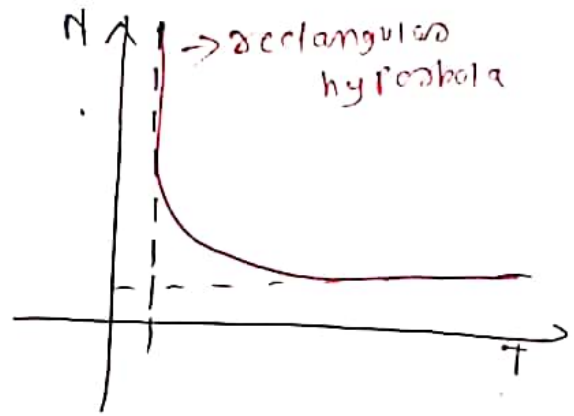
→ Elevators, Air compressors, Electric traction, cranes, Sewing machine.

Important

→ Series motor should never be started at no-load condition.

→ ~~Speed of Dc series~~

→ Direction of rotation of a Dc series motor can be changed by changing either field terminals or armature terminals.



compound motor characteristics

$N \propto \frac{E_b}{\phi_T}$        $\phi_T = \phi_{sh} \pm \phi_{se}$        $+$   $\rightarrow$  cumulative  
 $- \rightarrow$  differential

$\phi_{sh} \propto I_{sh} = \frac{V}{R_{sh}} = \text{constant}$        $\phi_{se} \propto I_a$

$T \propto \phi I_a$   
 $T \propto (\phi_{sh} \pm \phi_{se}) I_a$

$T_{em} \propto \phi_{sh} I_a \pm \phi_{se} I_a$

① N vs I<sub>a</sub>

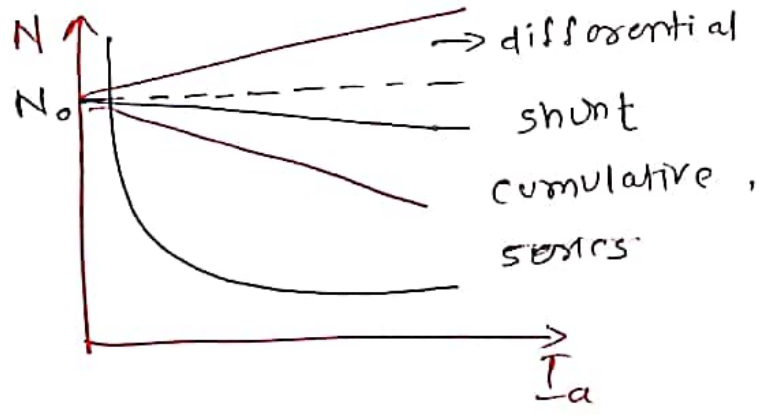
At No-load  $I_a \approx 0, \phi_{se} = 0$        $N \propto \frac{E_b}{\phi_{sh}} \propto \frac{V}{\phi_{sh}}$

$E_b \propto V - I_a(R_a + R_{se})$        $N = N_0$

$\phi_T$   
 load  $\uparrow, I_a \uparrow, \phi_{se} \uparrow$   
 cumulative

$\downarrow N \propto \frac{E_b}{(\phi_{sh} + \uparrow \phi_{se}) \uparrow}$

differential  
 $\uparrow N \propto \frac{E_b}{(\phi_{sh} - \phi_{se}) \downarrow}$



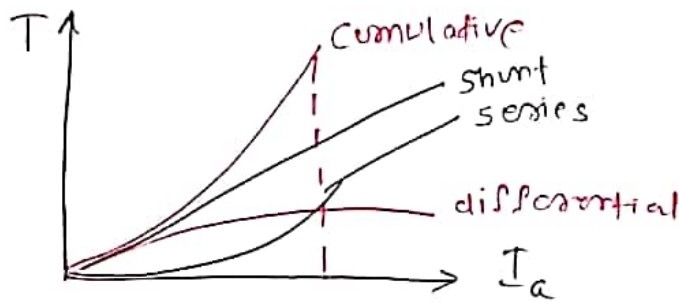
② T vs I<sub>a</sub>

$T \propto (\underbrace{\phi_{sh} I_a}_{\text{shunt}} \pm \underbrace{\phi_{se} I_a}_{\text{series}})$

At No-load,  $I_a = 0, T = 0$

load  $T = \text{constant}$   
 $\downarrow (\phi_{sh} - \phi_{se}) I_a \uparrow = \text{constant}$





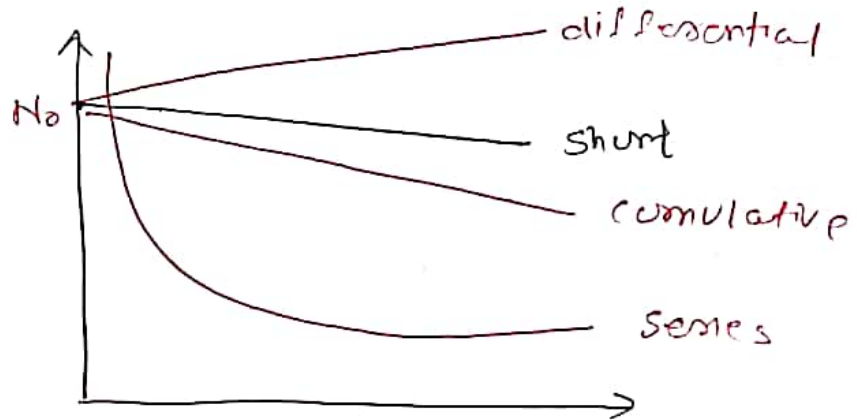
③ N VS T

At No-load

$$T=0, I_a=0, N=N_0$$

At load

$$T \uparrow, N \downarrow$$



Application

→ Moderate Torque at limited No-load speed

- |   |   |                      |
|---|---|----------------------|
| <ul style="list-style-type: none"> <li>i) punching machine</li> <li>ii) drilling machine</li> <li>iii) steel mills</li> <li>iv) cement mills</li> <li>v) paper mills</li> </ul> | } | cumulative compound. |
|---|---|----------------------|

→ differential compound m/cs - have very limited Application.

→ Speed regulation of shunt motor is +ve.

- " series motor is too. (poorest speed regulation)
- " cumulative compound is +ve,
- " differential " is -ve.

### Speed control of DC motors

#### Base speed or rated speed

→ It is the speed of the motor at rated supply voltage & at rated flux.

$N \propto \frac{V - I_a R_a}{\phi}$  or  $N \propto \frac{E_b}{\phi}$  → load on the machine = constant.

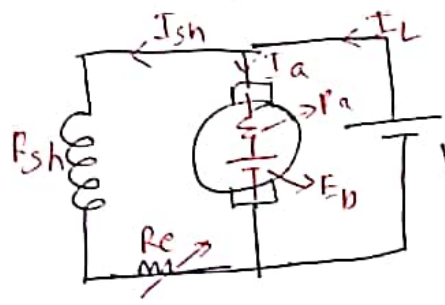
#### 3 methods of speed control

- i) Flux control method / field weakening method
- ii) Armature resistance control method
- iii) Armature voltage control method

#### shunt motor

#### ① Flux control method

$I_{sh} = \frac{V}{R_{sh} + R_e}$  and  $N \propto \frac{V - I_a R_a}{\phi}$



$(R_{sh} + R_e) \uparrow \rightarrow$  variable resistance is added in shunt field

$R_{sh} \uparrow, I_{sh} \downarrow, \phi \downarrow,$

$\phi \downarrow, N \uparrow$

→ In this method a variable resistance is placed in series with shunt field wdg.

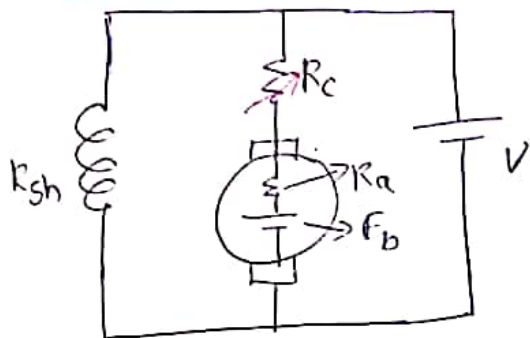
→ shunt field rheostat reduces the shunt field current  $I_{sh}$  & hence flux ( $\phi$ ) is reduced.

Hence speed is increased.



Flux control method is used for above rated speed.

② Armature Resistance control



$$\downarrow N \propto \frac{E_b}{\phi}$$

$$\downarrow N \propto \left[ V - I_a(R_a + R_c) \right] \downarrow$$

- This is done by inserting a variable resistance ( $R_c$ ) known as controller resistance.
- Due to extra drop in controller resistance ( $R_c$ ),  $E_b$  is decreased. Hence speed is reduced.
- The highest speed obtained when  $R_c = 0$  i.e. normal speed. Hence this method can provide speeds below the normal speed.

disadvantages

- i) A large amount of power is wasted in the  $R_c$ , since it carries full armature current  $I_a$ .
  - ii) o/p & efficiency of the motor reduced.
  - iii) This method results in poor speed regulation.
- Due to this disadvantage, this method is rarely used in case of DC shunt motor speed control.

③ Armature Voltage control method

Ward-Leonard speed control

- In this method, the motor is operating as separately excited motor.
- In this method, both above & below speed control is possible.

For below rated speed

- A variable voltage is applied to the armature motor by vary field of Generator -

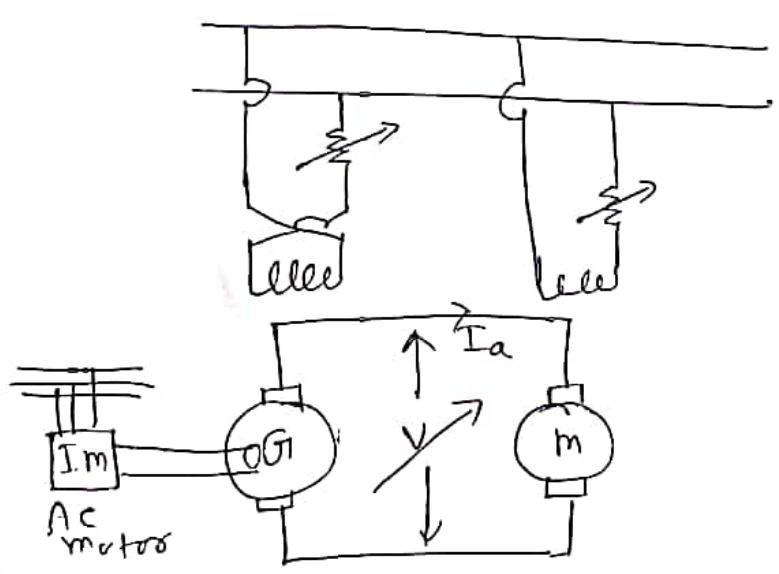


→ As voltage ↑,  $N \uparrow$ ,  $E_b \uparrow$

$V - E_b \approx \text{constant}$ .

⇒ Armature current & flux becomes constant.

∴  $T \propto \phi I_a$  is constant.



For above rated speed

→ Flux control method is used. By varying the field ~~generator~~ regulator of motor, the flux can be controlled to below a rated value there by speed can be controlled to above a rated value.

→ As  $\phi \downarrow$ ,  $N \uparrow$ ,  $T \downarrow$ .

Advantages

- ① The speed of the motor can be controlled in wide range.
- ② wide range of speed control possible in either direction.
- ③ Used for speed control of large motors.

disadvantages

→ High initial cost.

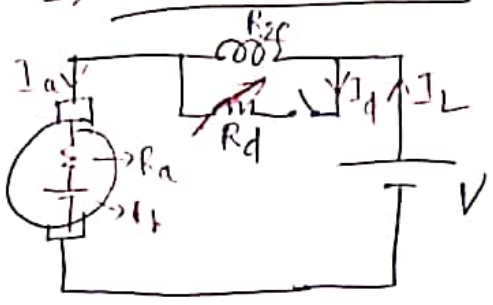
speed control of DC series motor

① FLUX control method

Flux of series motor can be controlled by using the following methods.

- i) diverter control
- ii) Tapped field control
- iii) series & parallel connection of field

1) Diverter control



without diverter

$I_{sc} = I_a$ ,  $N_1 = \text{rated speed}$

with diverter,  $I_{sc} \neq I_a$

$$I_{sc} = I_a - I_d = I_a \cdot \frac{R_d}{R_d + R_{sc}}$$

→ Now  $I_{sc}$  is less than  $I_a$  & flux will decrease.

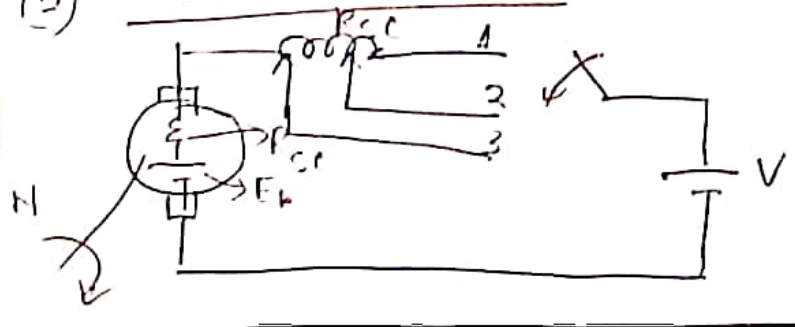
$$\frac{N_2}{N_1} = \frac{F_{b2}}{E_{b1}} \propto \frac{\phi_1}{\phi_2}$$

$$\Rightarrow \frac{N_2}{N_1} = \left\{ \frac{V - I_a [(R_{sc} \parallel R_d) + R_a]}{V - I_a (R_{sc} + R_a)} \right\} \times \frac{I_a}{I_{sc}}$$

→ In this method, a variable resistance called diverter resistance is connected in parallel with series field wdg.

→ It's effect is to shunt some portion of line current to field diverter. ( $N \propto \frac{1}{\phi}$ ). This method can only provide speeds above the normal speed.

② Tapped field control



A.L. ①  $\phi_1 \propto T_1$

②  $\phi_2 \propto T_2$

③  $\phi_3 \propto T_3$

$T_1 > T_2 > T_3$

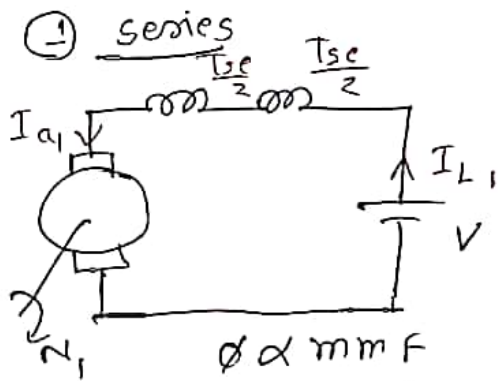
$\phi_1 > \phi_2 > \phi_3$

$N_1 < N_2 < N_3$

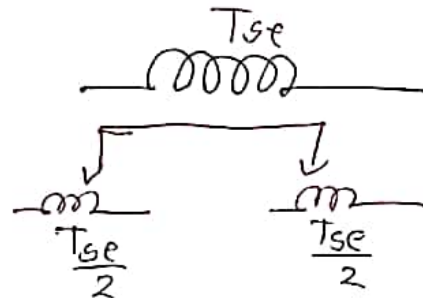
→ In this method, the flux is reduced hence speed is <sup>(21)</sup> increased by decreasing the number of turns of the series field winding.

→ With full turns of the field winding, the motor runs at normal speed and as the field turns are cut-out, speeds higher than the normal speed are achieved.

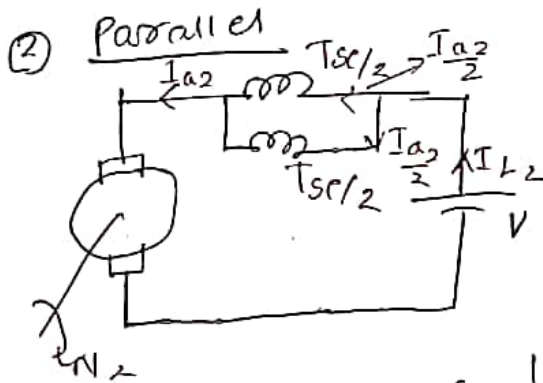
③ Series, Parallel connection



$$\phi \propto \text{MMF} = I_{a1} \frac{T_{se}}{2} + I_{a1} \frac{T_{se}}{2}$$



$$\phi_1 \propto I_{a1} T_{se}$$

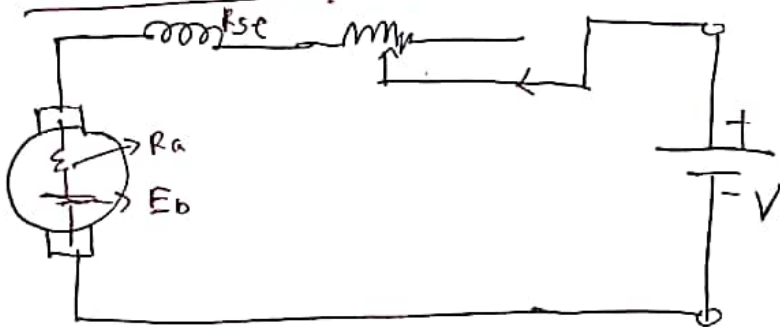


$$\phi_2 \propto \text{MMF} = \frac{T_{se}}{2} \times \frac{I_{a2}}{2} + \frac{T_{se}}{2} \times \frac{I_{a2}}{2}$$

$$\phi_2 \propto \frac{T_{se} I_{a2}}{2}$$

→ MMF becomes  $\frac{1}{2}$  & speed becomes double as compared to series connection.

④ Armature resistance control method



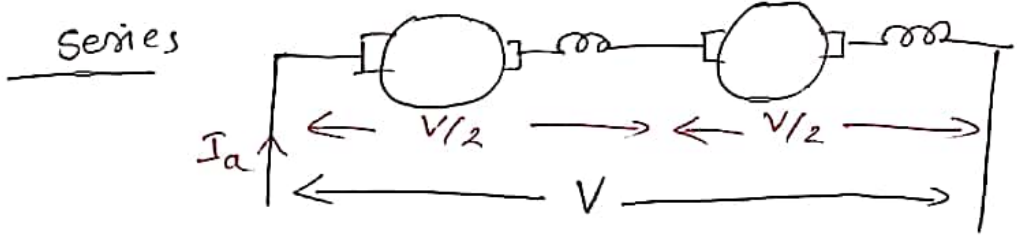
→ In this method, a variable resistance is directly connected in series with the supply to complete motor.

→ This reduces voltage available across the armature & hence speed falls.

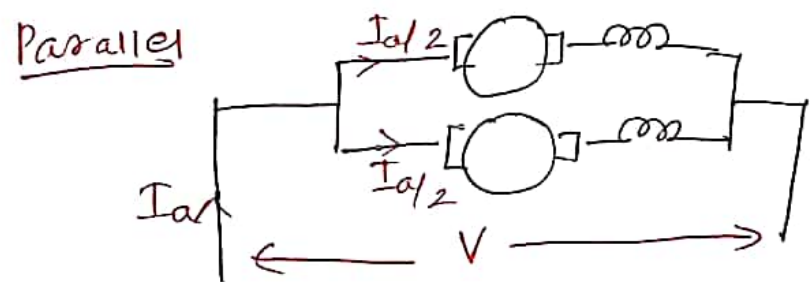


③ Armature voltage control method

→ series parallel method is used for speed control of DC series motor. In this method which is widely used in traction machine, two similar DC series motors are mechanically coupled to the same load.



$N_{se} \propto \frac{E_b}{\phi}$  in series  $\phi \propto I_a$   
 Neglecting  $I_a (R_a + R_{se})$  drop  
 $N_1 \propto \frac{V/2}{I_a}$        $N_1 \propto \frac{V}{2I_a}$  (ii)       $T_{se} \propto I_a^2$  (i)



$N_p \propto \frac{E_b}{\phi_2}$

$\phi_2 \propto (\frac{I_a}{2})$        $N_p \propto \frac{V}{(I_a/2)}$        $N_p \propto \frac{2V}{I_a}$  (iii)

$T_p \propto (\frac{I_a}{2})^2$  (iv)

$\frac{(iii)}{(i)} \Rightarrow \frac{N_p}{N_{se}} = \frac{2V}{I_a} \times \frac{2I_a}{V} = 4 \Rightarrow N_p = 4 N_{series}$

$\frac{(iv)}{(ii)} \Rightarrow \frac{T_p}{T_{se}} = \frac{(\frac{I_a}{2})^2}{(I_a)^2} = \frac{I_a^2}{4 I_a^2} \Rightarrow T_p = \frac{1}{4} T_{se}$

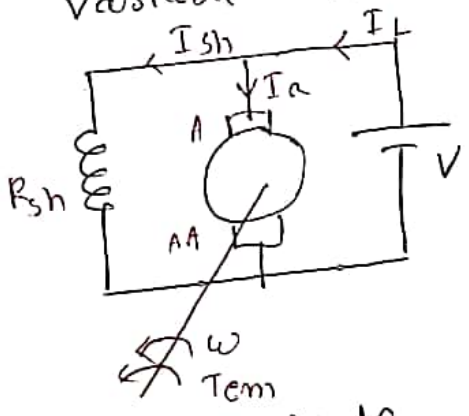
$T_{se} = 4 T_p$   
 ↓ Traction

# Electric braking

- It is of '3' types .
  - Dynamic braking or Rheostat braking
  - plugging (or) reverse current braking
  - Regenerative braking

## ① Dynamic braking or Rheostatic braking

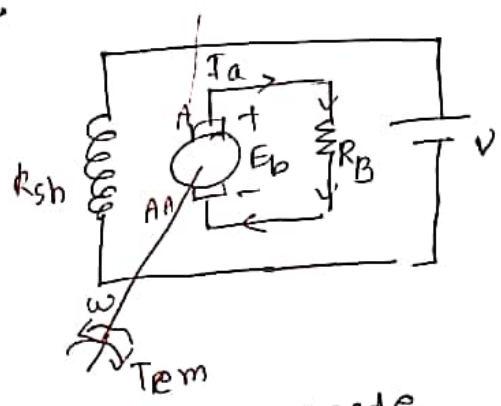
→ In this method, the armature of the running motor is disconnected from the supply & is connected across a variable resistance R.



motoring mode

$T_{em} \propto \phi I_a$

⇒ torque direction counter clockwise.



braking mode

$T_{em} \propto \phi (-I_a)$

⇒ torque direction ~~counter~~ clockwise.

→ In braking mode, motor acts as a generator.

Braking current,  $I_{aB} = \frac{E_b}{R_a + R_B}$

$T_B \propto N$

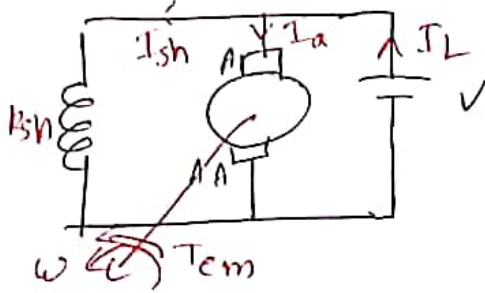
∴ Therefore braking torque decreases as the motor speed decreases.

→ This type of braking is used extensively in connection with the control of elevators & hoists and in other applications in which the motor must be started, stopped & reversed frequently.

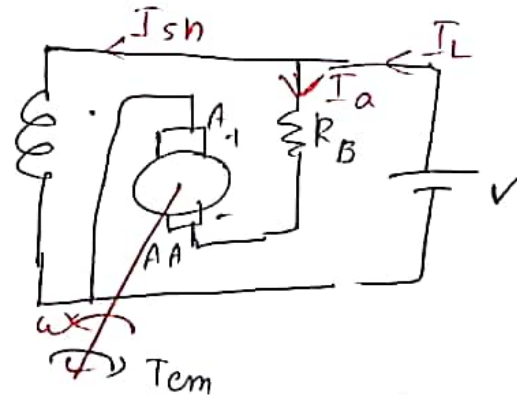
## ② Plugging

24

- In this method, connections of the armature are reversed so that motor tends to rotate in the opposite direction, thus necessary braking is provided.
- when the motor comes to rest, the supply must be cut off otherwise the motor will start rotating in the opposite direction.



motoring mode  
→ ccw rotating



→ braking mode  
→ cw rotation

- Due to reversal of armature connections, applied voltage  $V$  &  $E_b$  start acting in the same direction around the circuit.

→ braking current  $I_{ab} = \frac{V + E_b}{R_a + R_B}$

- Here  $R_B$  is required to limit the armature current.

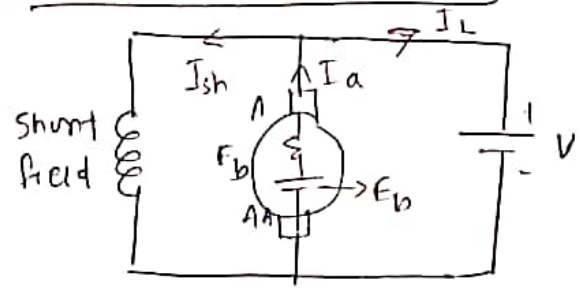
⇒  $T_B = k_1 + k_2 N$

- During plugging the electromagnetic torque opposes the rotation. Then speed will be reduced gradually to zero.

- Before coming to zero, supply has to be disconnected and mechanical brakes are applied.



### ③ Regenerative braking



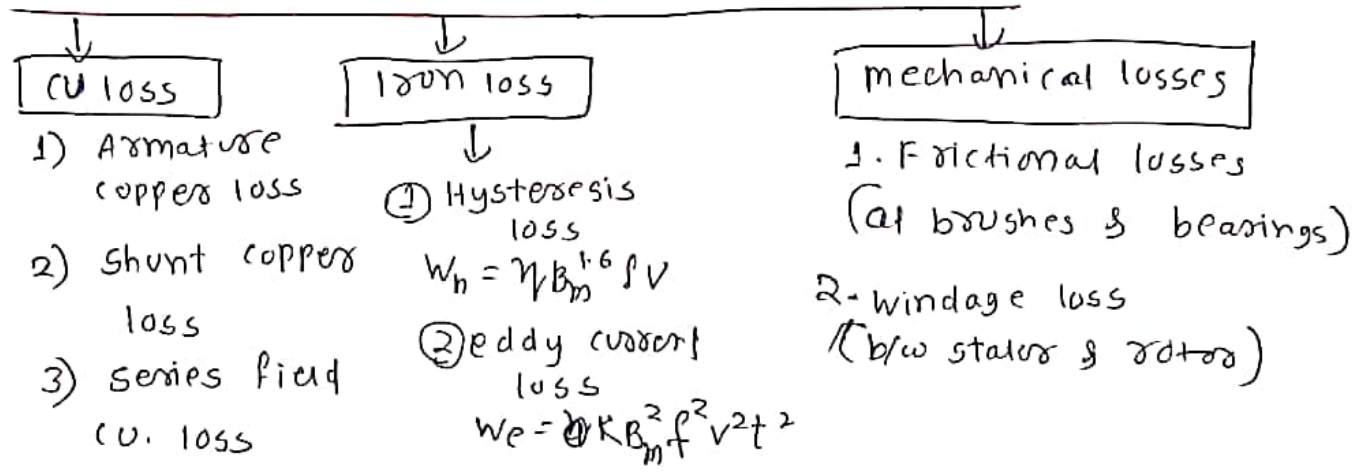
Regenerative braking

→  $E_b < V$  → motoring operation

→  $E_b > V$  → Regenerative braking operation

→ In motoring operation, motor consumes the power from busbar. if  $E_b > V$ , then power flow is reversed i.e. direction of armature current  $I_a$  is reversed, so torque will reverse, braking action takes place.

### losses in DC Machine



### Constant losses

→ The losses which remains constant at all loads are known as constant losses.

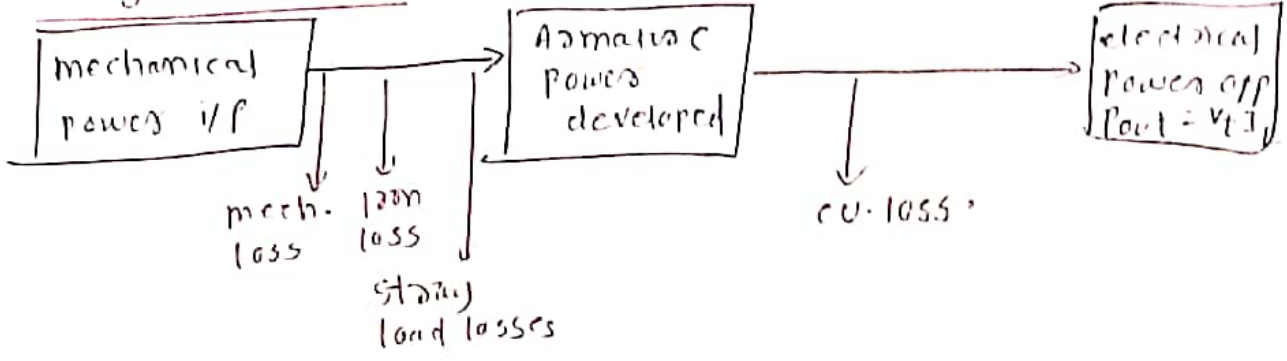
- ① Iron loss
- ② mechanical loss
- ③ shunt field losses

Variable loss ÷ These losses vary with load are called variable loss.

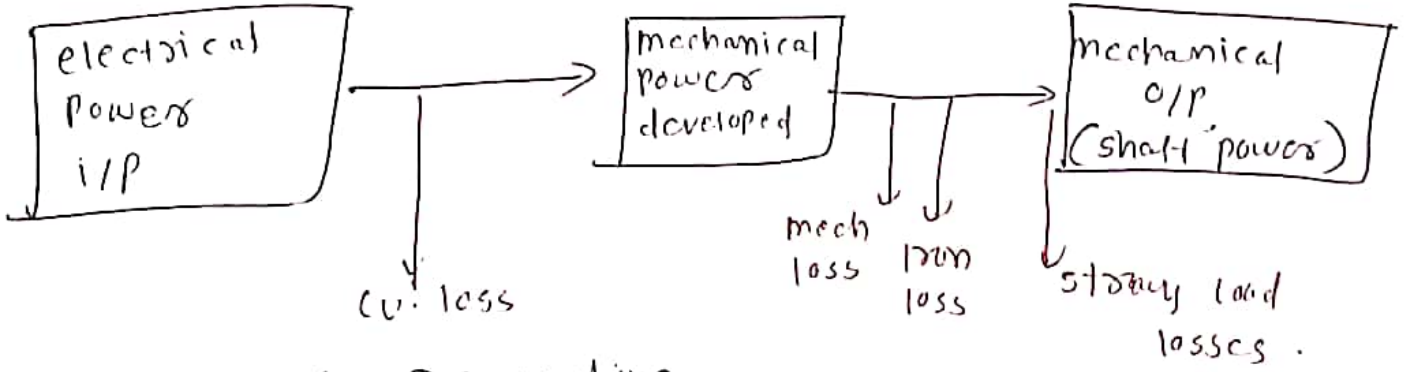
- ① Cu. loss in armature winding
- ② Cu. loss in series winding.

Power stages

In generator



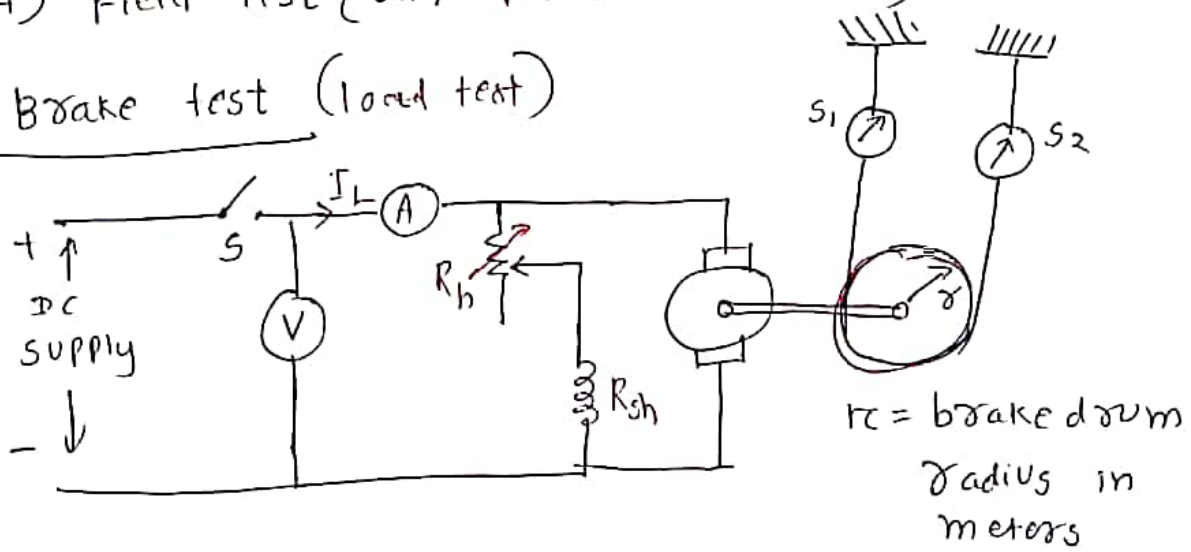
In motor



Testing of DC machine

- 1) Brake test / direct load test
- 2) Swinburne's test / No-load test / indirect load test
- 3) Hopkinson's test / Regenerative / back to back test
- 4) Field test (only for series motors)

Brake test (load test)



- To determine directly the efficiency of comparatively <sup>(27)</sup> small motors, the motor is loaded directly by means of a mechanical brake.
- Brake test is a direct method i.e. the motor or generator put on full load and whole of the power developed by it is wasted.
- This method is not suitable for large rating machine, because more frictional losses at brake drum.

$$P_{in} = V I_L, \quad P_o = \omega \times T = \frac{2\pi N T}{60} \text{ watt}$$

$$T_{sh} = F \times (\text{perpendicular distance})$$

$$T_{sh} = mg \times r$$

$$T_{sh} = (s_1 \sim s_2) \times 9.81 \times r$$

$s_1, s_2$  = reading of spring balance in kg.

$r$  = radius of brake drum

$$\text{efficiency } \eta = \frac{P_{out}}{P_{in}} \times 100$$

$$P_{out} = \frac{2\pi N T_{sh}}{60}$$

### Advantage

→ machine performance is checked at actual loaded cond<sup>n</sup>.

### disadvantage

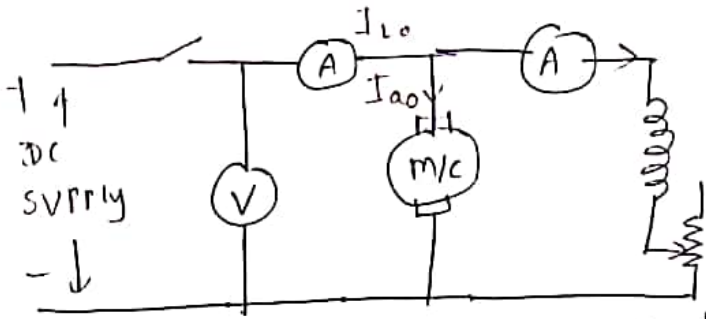
→ Total power i/p is wasted. Hence it is uneconomical & not suitable for large DC machine.

→ Spring balance reading are not studied.

## ② Swinburne's test (No-load test)

In this method the losses are calculated, then efficiency is calculated. Therefore this test is indirect test.





→ since this is a no-load test, it is not suitable for series motors because under no load condition it rotate with dangerously high speed.

(X) This test is suitable for constant flux machine (shunt & compound)

→ This machine will be operated as a motor though it is a generator.

→ The m/c is operated under no-load cond<sup>n</sup>. Therefore the constant losses are measured and with the knowledge of constant losses the efficiency can be predetermined at any desired load condition.

$$P_{in} = P_{out} + \text{losses}$$

$$\boxed{\text{No load} \Rightarrow P_{out} = 0}$$

$$V I_{L0} = 0 + \text{losses}$$

$$V I_{L0} = \text{losses (cu-loss + constant loss)}$$

$$V I_{L0} = I_{a0}^2 R_a + W_c$$

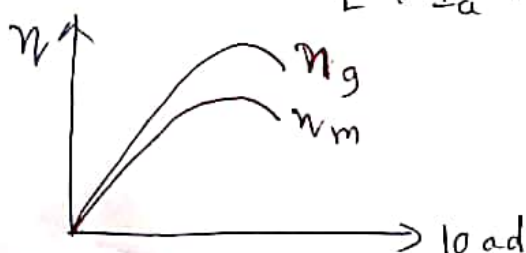
$$\Rightarrow \boxed{W_c = V I_{L0} - I_{a0}^2 R_a}$$

For Generator  $\eta = \frac{P_o}{P_{in}} = \frac{P_o}{P_o + \text{losses}} \times 100$

$$\% \eta_{FL} = \frac{V I_L}{V I_L + I_a^2 R_a + W_c}$$

For motor  $\rightarrow \frac{P_{out}}{P_{in}} = \frac{P_{in} - \text{losses}}{P_{in}}$

$$\% \eta_{FL} = \frac{V I_L - I_a^2 R_a - W_c}{V I_L}$$



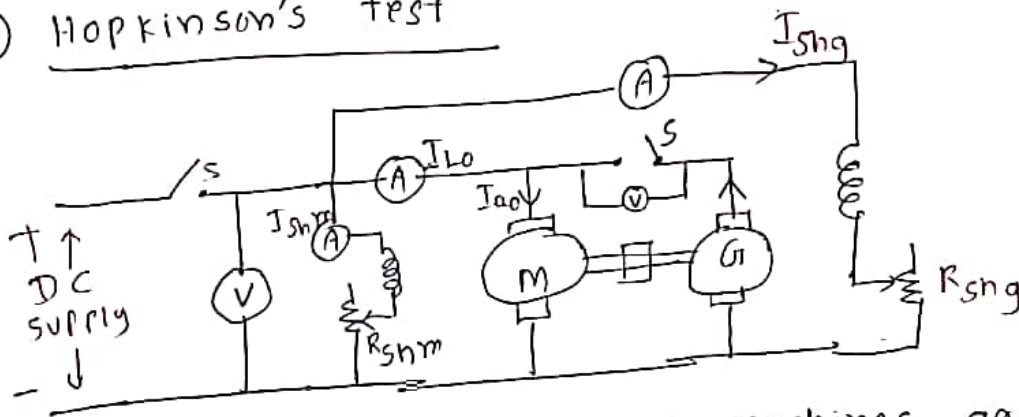
## Advantages

- i) The power drawn from the supply is only to meet the losses. So experiment is economical.
- ii) Large rating of machine can also be tested.

## disadvantage

- i) Machine performance is not checked at actual loaded cond<sup>n</sup> i.e. the effect of Armature reaction, commutation & temperature rise are not considered.
- ii) efficiency is more as stray load losses are not considered.
- iii) not suitable for series motors.

## ③ Hopkinson's test



- For this test 2 identical machines are required.
- Both the machines are connected mechanically & electrically coupled. one of them works as a motor & another one is generator.
- load on the motor is generator & load on generator is motor. power drawn from the supply is only for supplying internal losses of two machines.
- This test is a Regenerative test for determining efficiency of machines.

- operation when the supply is given, the motor rotates & its speed is adjusted to rated value.
- since the Generator is mechanically coupled to motor, the generator also rotates & generates its voltage. since field winding is connecting to supply adjust the field regulator of generator such that voltmeter reads zero. Then both are said to be at same Potential & polarity.
- By closing the switch across the voltmeter, the two machines are connected in parallel but no power flow b/w the two. It is said to be floating on the bus bars.
- The field of the generator is strengthened and simultaneously field of motor is weakened so that  $I_a$  is adjusted to its rated value. The o/p of the generator is given to motor & mechanical power o/p of motor is given to the generator, the two machines are exchanging their powers. The load on the generator is motor & load on the motor is generator.
- If there is no loss in the machines, the power drawn from supply is zero. But to compensate the loss it will draw the power is  $VI_{Lo}$ .
- stray ~~losses~~ losses are measured, equally divided both for Generator & motor.

$$VI_{Lo} = \text{Total losses in both machines excluding shunt copper losses.}$$

$$= I_{ag}^2 R_{ag} + I_{am}^2 R_{am} + \text{Stray losses in both machines.}$$



$$\text{Stator losses } (W_s) = VI_{Lc} - I_{ag}^2 R_{ag} - I_{am}^2 R_{am} \quad (31)$$

$$\text{Stator loss for each machine} = \frac{W_s}{2}$$

efficiency can be determined

Generator

$$\eta_g = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + \text{losses}}$$

$$= \frac{VI_{ag}}{VI_{ag} + I_{ag}^2 R_{ag} + I_{shg}^2 R_{shg} + \frac{W_s}{2}} \times 100$$

Motor

$$\eta_m = \frac{P_{out}}{P_{in}} = \frac{P_{in} - \text{losses}}{P_{in}}$$

$$\eta_m = \frac{V(I_{am} + I_{shm}) - I_{am}^2 R_m - I_{shm}^2 R_{shm} - \frac{W_s}{2}}{V(I_{am} + I_{shm})} \times 100$$

Advantages

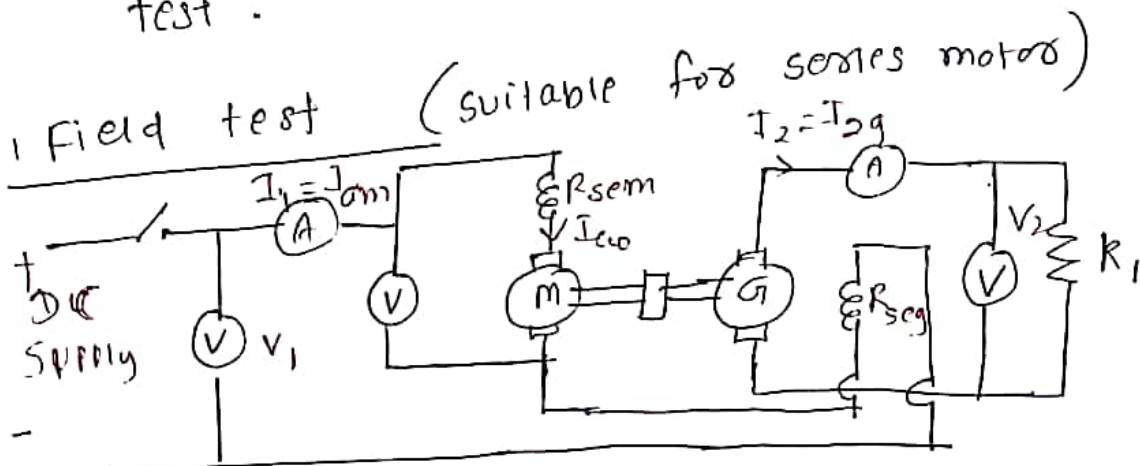
i) machine performance is checked at actual loaded condition. Therefore effect of AR, commutation & temp. rise is considered.

ii) This test is economical, because power required to meet the losses in both the M/Cs.

Disadvantages

i) Two identical M/Cs are required.

ii) Equal division of stray load losses are not justified because generator field current is greater than motor field current in Hopkinson's test.



- (32)
- This test is suitable for series machine. Two identical machines are required, one acting as generator & other one as motor.
  - Both are mechanically coupled but electrically isolated. The field winding of generator is connected in motor circuit so that the stray load losses can be equally divided for two machines.
  - In this test stray losses are measured & equally divided for two m/cs.

### Advantages

- Actual performance of the machine is verified.
- Stray load losses are considered & they are equally divided which is justified.

### disadvantage

- Two identical m/cs are required.
- The entire power drawn from supply is wasted across load resistance.

### Starters

- Function of starter is to limit the high starting current.

→  $I_a = \frac{V - E_b}{R_a}$  At the time of starting  $N=0$   
 $E_b=0$

$I_{st} = \frac{V}{R_a}$  is very high due to absence of opposing emf or back emf.

by using a starter,  $I_{st}$  becomes as  $I_{st} = \frac{V}{R_a + R_s}$

- $I_{st}$  decreases.

Types of starters

(33)

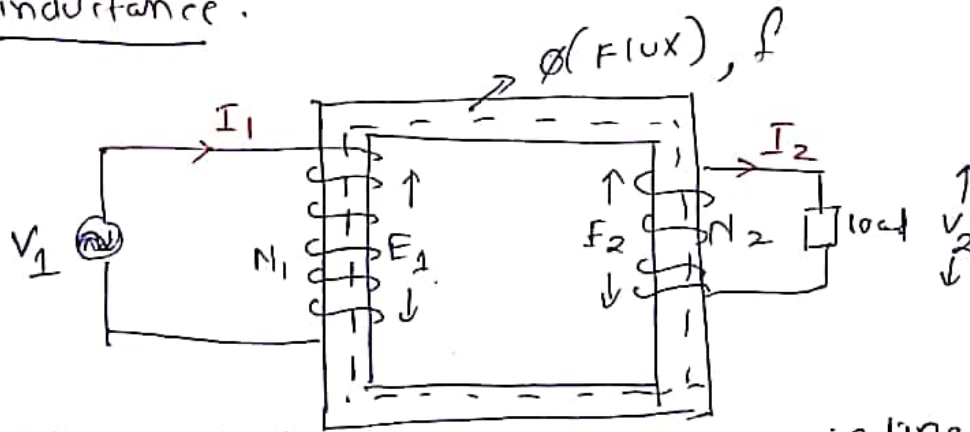
- ① 3-point starters
- ② 4-point starters



# Transformers

## Working principle

- A transformer is a static device by means of which electric power in one circuit is transferred into electric power into another circuit ~~with~~ without change in frequency.
- Transformer works on the principle of mutual inductance.



→ It essentially consists of two windings, the primary & secondary.

$V_1$  = Applied input voltage

$V_2$  = load voltage / output voltage

$N_1$  = No. of primary turns

$N_2$  = No. of secondary turns

$E_1$  = self induced emf

$E_2$  = mutually induced emf

$\phi_m$  = Maximum flux in core in webers

$f$  = frequency in AC input in Hz

## Working principle

- When an alternating voltage  $V_1$  is applied to the primary, an alternating flux ( $\phi$ ) is set up in the core. This alternating flux links both the windings & induces e.m.f's  $E_1$  &  $E_2$  in them

according to Faraday's laws of electromagnetic induction, The emf  $E_1$  is termed as primary emf &  $E_2$  is termed as secondary emf.

$$E_1 = -N_1 \frac{d\phi}{dt}, \quad E_2 = -N_2 \frac{d\phi}{dt} \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = k \longrightarrow \text{Voltage transformation ratio}$$

- i) if  $N_2 > N_1$ , i.e.  $k > 1 \rightarrow$  step-up transformer  
 ii) if  $N_2 < N_1$ , i.e.  $k < 1 \rightarrow$  step-down transformer

turn's ratio (a)

$$a = \frac{N_1}{N_2} = \frac{E_2}{E_1} = \frac{1}{k}$$

For an ideal transformer  $V_1 I_1 = V_2 I_2$   
 $\Rightarrow \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{k}$   
 Input VA = O/P VA

Transformer core  $\rightarrow$  cold rolled grain oriented silicon steel (CRGO steel) with lamination

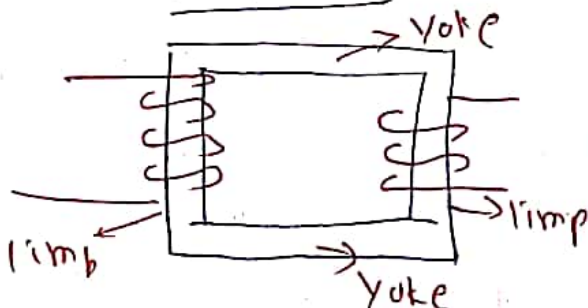
Transformer winding  $\rightarrow$  Copper (For power transformer)  
 Aluminium (For distribution transformer)

Types of core

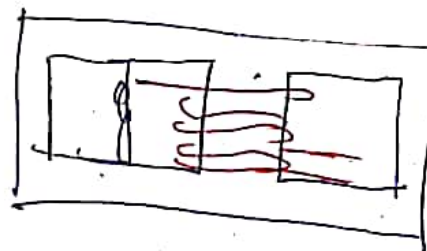
1) core type  $\rightarrow$  2 yoke, 2 limbs

2) shell type  $\rightarrow$  2 yoke, 3 limbs

core type



shell type



## Core type

i) Windings placed on both the limbs & core is surrounded by windings. Hence protection to core is less.

ii) All limbs have same cross-sectional area.

iii) Analogous to series magnetic circuit.

iv) Leakage flux is more.

v) Less insulating material & more copper is required.

vi) High voltage, small KVA rating T/F.

## Shell type

i) Windings are surrounded by core, hence protection is more to winding.

ii) Outer limbs are half of middle limb cross section.

iii) Analogous to parallel magnetic circuit.

iv) Leakage flux is less.

v) More insulation & less copper is required.

vi) Low voltage, high KVA rating T/F.

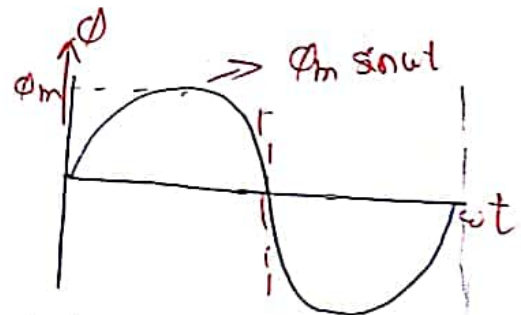
## E.m.f eqn of Transformer

→ Consider that an alternating voltage  $V_1$  of frequency  $f$  is applied to the primary.

→ The sinusoidal flux ( $\phi$ ) produced by the primary is  $\phi = \phi_m \sin \omega t$

$$\begin{aligned} E_1 &= -N_1 \frac{d\phi}{dt} = -N_1 \frac{d(\phi_m \sin \omega t)}{dt} \\ &= -\omega N_1 \phi_m \cos \omega t \\ &= -2\pi f N_1 \phi_m \cos \omega t \\ &= 2\pi f N_1 \phi_m \sin(\omega t - 90^\circ) \end{aligned}$$

$$E_{1 \max} = 2\pi f N_1 \phi_m, \text{ RMS value } E_{1 \text{ RMS}} = \frac{E_{1 \max}}{\sqrt{2}}$$

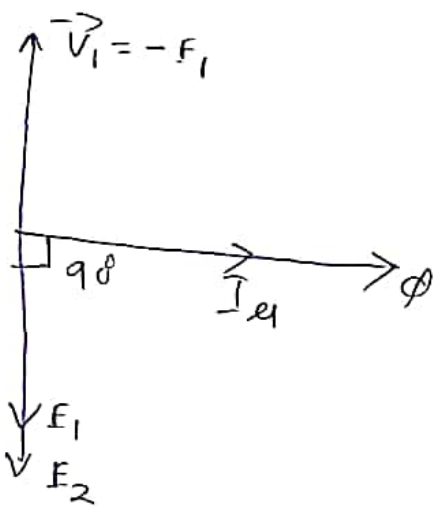




$E_1 = 4.44 f N_1 \phi_m$   
 similarly  $E_2 = 4.44 f N_2 \phi_m$

$\phi_m = B_m A_n$

$\Rightarrow E_1 = 4.44 f N_1 B_m A_n$   
 (rms)



Transformers at No-load

The No load primary current ( $I_0$ ) is divided into 2 types.

magnetizing component of current ( $I_e$ )

$\rightarrow$  required to produce flux in transformer core.

iron loss component ( $I_w$ )

$\rightarrow$  to meet the iron losses at No. load.

so  $\vec{I}_0 = \vec{I}_e + \vec{I}_w$

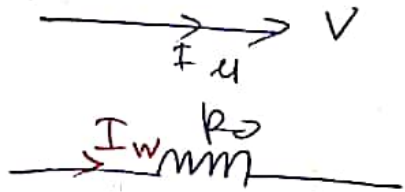
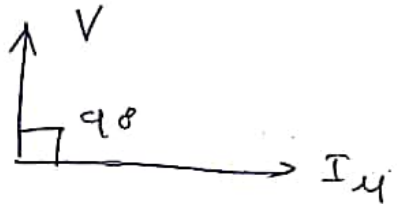
i.e.  $I_0 = \sqrt{I_e^2 + I_w^2}$

$\rightarrow$  Analogous to reactive power component or wattless component

$\rightarrow$  Analogous to active power component or wattful component.

$\rightarrow I_e$  lags 'V' by  $90^\circ$ .

$\rightarrow I_w$  in phase with 'V'.

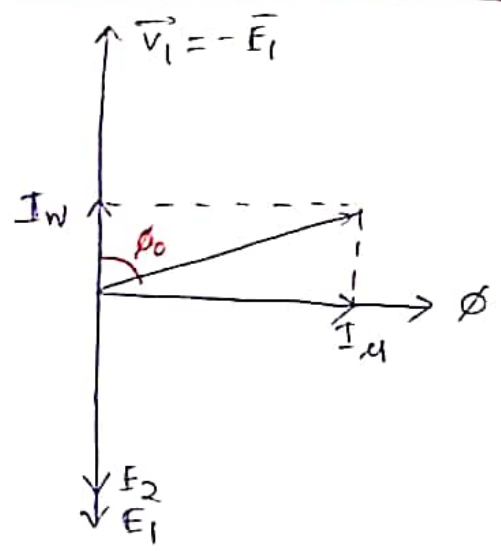


$I_e$   $\approx 4$  to  $6\%$  of  $I_{FL}$

$\rightarrow 1$  to  $2\%$  of  $I_{FL}$

$I_M \gg I_W$

T/F Phasor diagram under No-load cond<sup>n</sup>



$\phi_0 =$  No-load phase angle of T/F  
 $\phi_0 = 70 \text{ to } 75^\circ$   
 $\cos \phi_0 =$  No-load Power Factor  
 $= 0.2 \text{ lag}$

$I_W = I_0 \cos \phi_0$   
 $I_M = I_0 \sin \phi_0$

No-load power consumption of T/F =  $V_1 I_0 \cos \phi_0$

$\Rightarrow W_0 = V_1 I_W = \text{iron loss only}$

Ideal Transformer

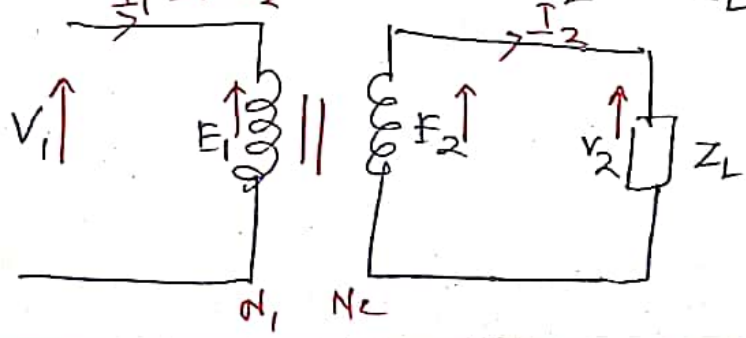
An ideal transformer is that which has

- i) no winding resistance
- ii) no leakage flux i.e. the same flux links both the windings
- iii) no iron losses (i.e. eddy current & hysteresis loss) in the core

Ideal T/F on load

→ Let us connect a load  $Z_L$  across the secondary of an ideal transformer. The secondary emf  $E_2$  will cause a current  $I_2$  to flow through

the load.  $I_2 = \frac{E_2}{Z_L} = \frac{V_2}{Z_L}$



→ we have considered inductive load so that current  $I_2$  lags behind  $V_2$  or  $E_2$  by  $\phi_2$ .

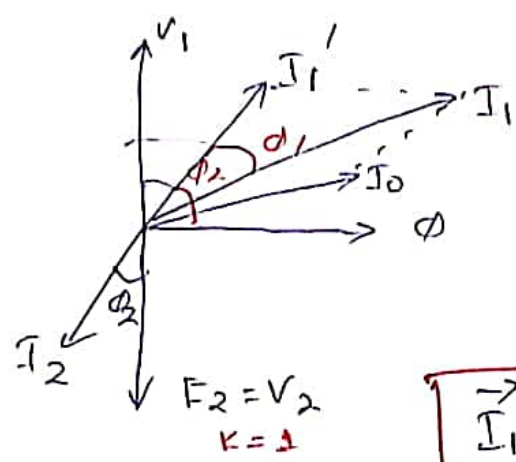
→  $I_2$  sets up an mmf  $N_2 I_2$  which produces a flux in the opposite direction to the flux  $\phi$  originally set up in the primary. This reduces the net flux in the core.

→ In order to minimise the reduction, the primary draws an additional current  $I_1'$  which must develop an mmf  $N_1 I_1'$  which counterbalances the secondary mmf  $N_2 I_2$ .

so  $N_1 I_1' = N_2 I_2 \Rightarrow \boxed{I_1' = k I_2}$

$\Rightarrow I_1' = \frac{N_2}{N_1} I_2$

Hence flux in the core remains constant.



$I_1'$  = load component of primary current

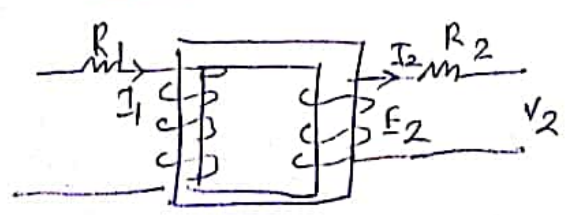
$I_0$  = no-load primary current

$I_1$  = primary current at load

$\vec{I}_1 = \vec{I}_0 + \vec{I}_1'$

$\boxed{I_1 = \sqrt{I_0^2 + I_1'^2}}$

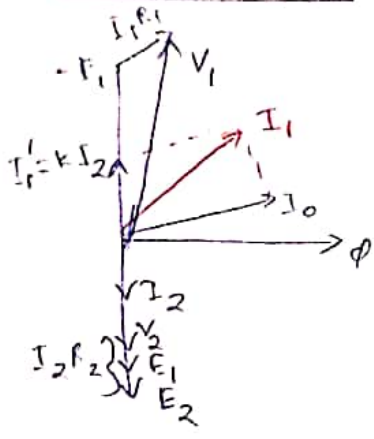
Transformer with winding resistance but no magnetic leakage reactance



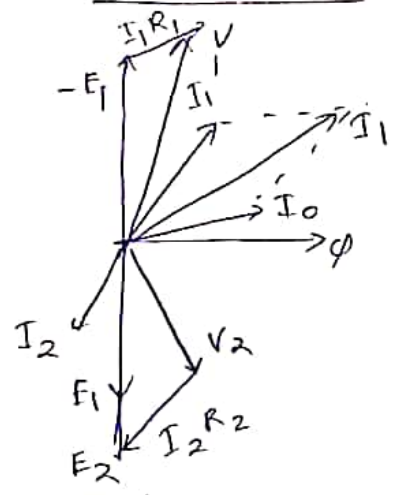
$V_2 = E_2 - I_2 R_2$   
 $E_1 = V_1 - I_1 R_1$  } due to presence of winding resistance.



Resistive load

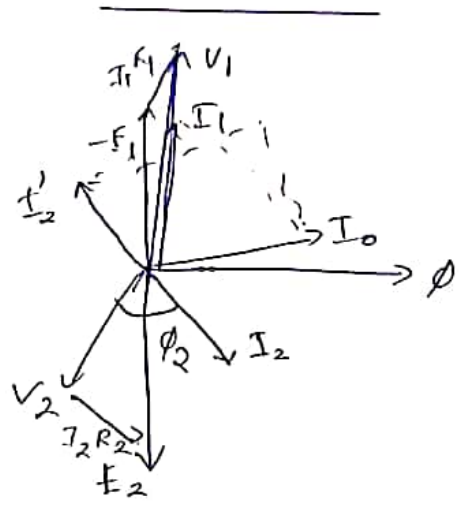


Inductive load



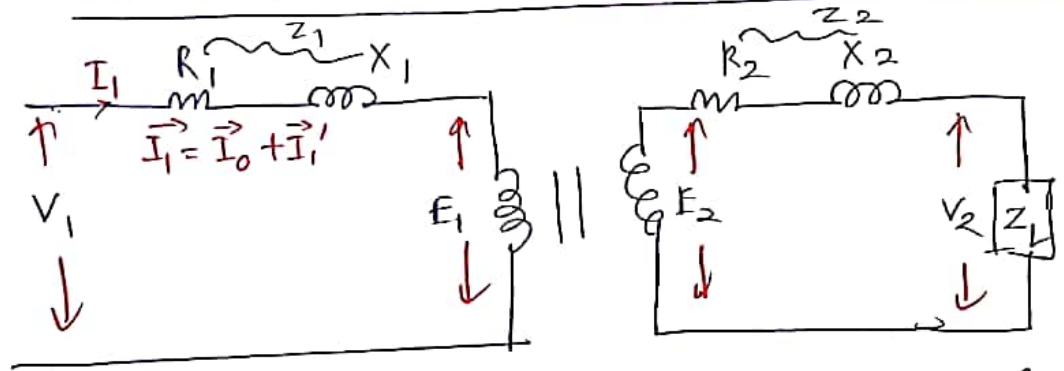
capacitive load

(7)



~~Transformer with resistance & leakage reactance~~

Transformer with resistance & leakage reactance



$$V_1 = -E_1 + I_1(R_1 + jX_1)$$

$$= -E_1 + I_1 Z_1$$

$$V_2 = E_2 - I_2(R_2 + jX_2)$$

$$= E_2 - I_2 Z_2$$

For inductive load

$\phi_0 = \text{N.L. P.F. angle}$

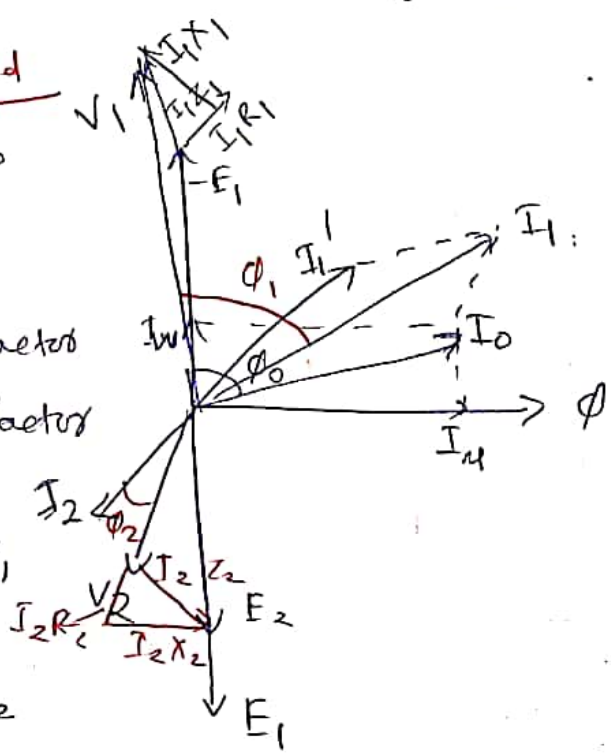
$\cos \phi_0 = \text{N.L. Power Factor}$

$\cos \phi_2 = \text{load power factor}$

$\cos \phi_1 = \text{i/p power factor}$

i/p power to T/F =  $V_1 I_1 \cos \phi_1$

o/p power of T/F =  $V_2 I_2 \cos \phi_2$



Impedance ratio

$$\frac{R_2}{R_1} = k^2, \quad \frac{X_2}{X_1} = k^2, \quad \frac{Z_2}{Z_1} = k^2$$

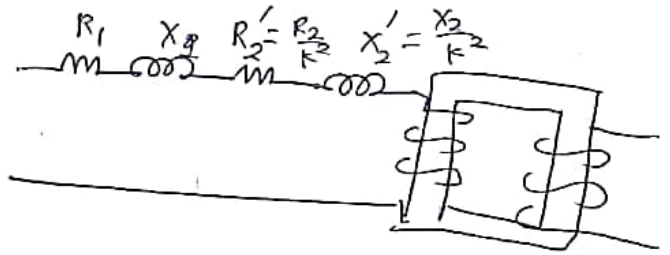
(A) shifting to primary or referred to primary

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{k^2}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{k^2}$$

equivalent impedance of T/F referred to primary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$



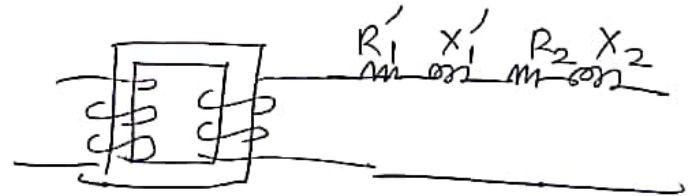
(B) shifting to secondary or referred to secondary

$$R_{02} = R_2 + R_1' = R_2 + k^2 R_1$$

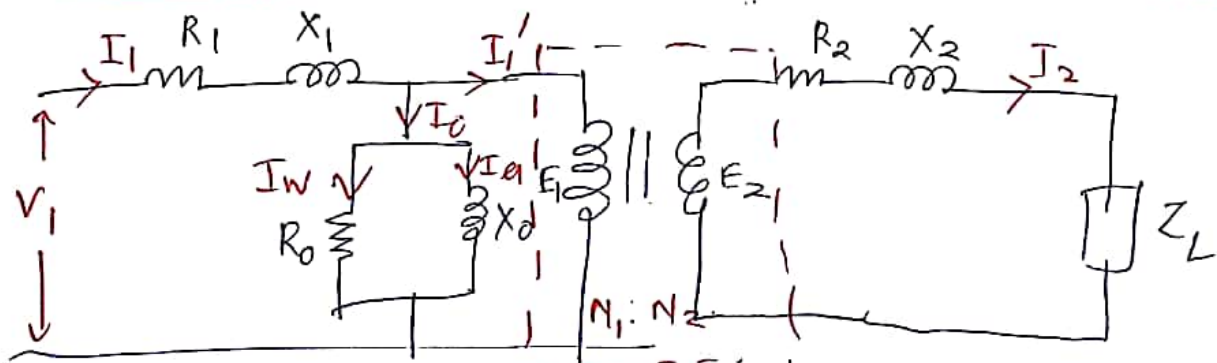
$$X_{02} = X_2 + X_1' = X_2 + k^2 X_1$$

equivalent impedance of T/F referred to 2°

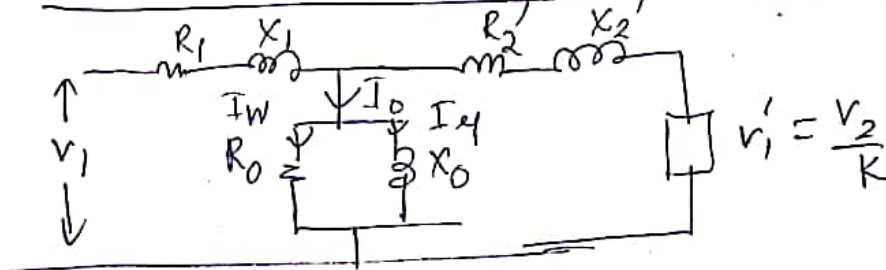
$$\Rightarrow Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$



Exact equivalent circuit of a loaded transformer

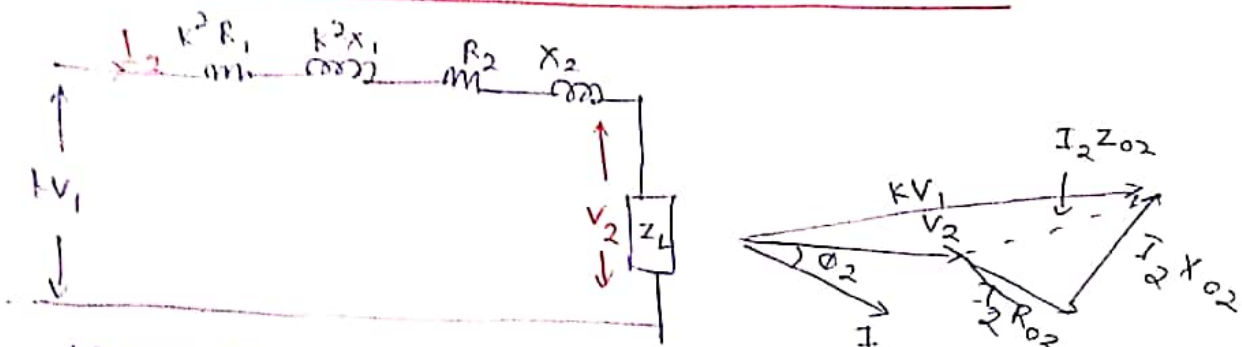


equivalent circuit of transformer refer to 1°



## Approximate voltage drop in transformer

(9)



Approximate drop in secondary voltage

$$= I_2 R_{02} \cos \phi_2 \pm I_2 X_{02} \sin \phi_2 \quad + \rightarrow \text{lagging P.F. load}$$

-  $\rightarrow$  leading P.F. load

## Voltage regulation

The voltage regulation of a transformer is the arithmetic difference b/w the no-load secondary voltage ( $0V_2$ ) & the secondary voltage  $V_2$  on load, as expressed as a percentage of no-load voltage,

$$\gamma. V. R = \frac{0V_2 - V_2}{0V_2} \times 100 \rightarrow \text{V.R. is best for lagging P.F. loads}$$

## Losses in Transformer

The power losses in a Transformer are of two types

(1) core or iron loss

(1) core or iron loss

$\rightarrow$  These consists of hysteresis & eddy current losses & occur in the transformer core due to alternating flux.

$$\rightarrow \text{Hysteresis loss } = (W_h) = \eta b_{\max}^{1.6} f V$$

where  $\eta$  = Steinmetz constant

$f$  = frequency of supply

$V$  = volume of core.



Transformer at any P.F. (10)

$$\text{Eddy current loss} \Rightarrow W_e = k_e B_m^2 f^2 V^2 t^2$$

where  $t$  = thickness of lamination

$B_m$  = max flux density

$f$  = frequency of operation

$k_e$  = eddy-current co-efficient

(value depends on magnetic material)

→ These losses are minimised by using steel of very high Silicon content for the core & by using very ~~high~~ thin laminations.

② Copper loss This loss is due to the ohmic resistance of the transformer windings.

$$\begin{aligned} \rightarrow \text{Total cu. loss} &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_1^2 R_{01} \text{ or } I_2^2 R_{02} \end{aligned}$$

∅ Total losses in Transformer =  $W_i + W_{cu}$   
 $W_i$  → iron loss or constant loss  
 $W_{cu}$  → copper loss or variable loss

### Efficiency of Transformer

$$\begin{aligned} \text{Efficiency} &= \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}} \\ &= \frac{\text{output}}{\text{output} + \text{cu. loss} + \text{iron losses}} \end{aligned}$$

$$\eta = \frac{\text{input} - \text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{\text{input}}$$

~~Cond<sup>n</sup> for max<sup>n</sup>~~

Full-load iron loss =  $W_i$  → from open circuit test

Full-load cu. loss =  $W_{cu}$  → from short circuit test

$$\text{Total F.L. losses} = W_i + W_{cu}$$

Full-load efficiency of the transformers at any p.f.

$$\eta_{F.L} = \frac{\text{Full-load VA} \times \text{p.f.}}{(\text{Full-load VA} \times \text{p.f.}) + W_i + W_{cu}}$$

$$= \frac{E_2 I_2 \cos \phi_2}{E_2 I_2 \cos \phi_2 + W_i + W_{cu}} \rightarrow \text{From s.c. test}$$

$\downarrow$   
 From d.c test

For any load equal to  $x$  of F.L.

$$\eta_x = \frac{x E_2 I_2 \cos \phi_2}{x E_2 I_2 \cos \phi_2 + W_i + x^2 W_{cu}}$$

cond<sup>n</sup> for max<sup>m</sup> efficiency

$$W_i = W_{cu} \quad \left[ \text{cu. loss} = \text{Iron loss} \right]$$

o/p KVA ~~is~~ corresponding to maximum efficiency  
 $= \text{Full-load KVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. cu loss}}}$

All day efficiency

$$\eta_{\text{all day}} = \frac{\text{o/p in kWh}}{\text{i/p in kWh}} \quad (\text{For 24 hours})$$

- All day efficiency is less than commercial efficiency.
- It is used for distribution transformers whose secondary is subject to variable losses.

Q) Why transformer rating in kVA?

A: cu. loss of a T/F depends on current  
 iron loss " " " " voltage.  
 Total T/F loss depend on volt-ampere (VA) & not phase angle b/w voltage & current, i.e. it is independent of Power Factor. So T/F rating is in kVA not in kW.

# Transformers tests ÷ 2 tests

i) open circuit test

ii) short circuit test

## ① open circuit or no-load test

This test is used to determine

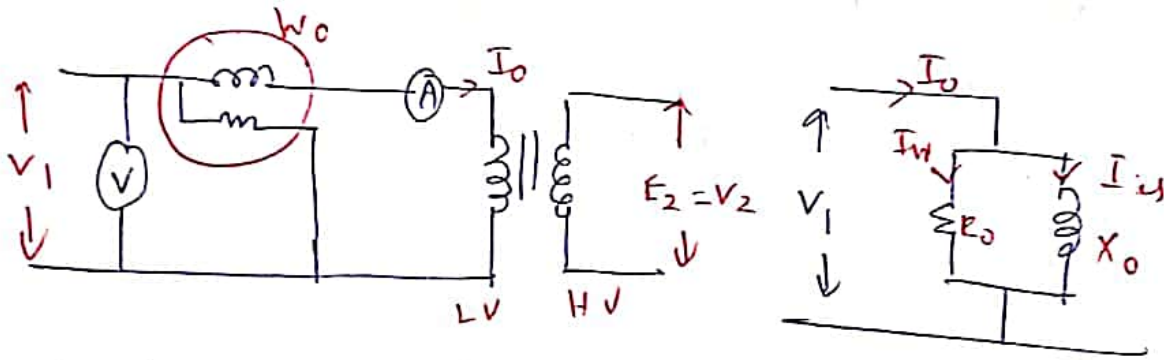
i) iron loss or core losses or constant losses

ii)  $R_0$  &  $X_0$  (shunt branch parameters)

iii) separation of iron loss into  $W_h$  &  $W_e$

→ In this method, the rated voltage is applied to the low voltage side i.e. primary & secondary is left open circuited.

→ The i/p voltage  $V_1$  is measured by the voltmeter, the no-load current  $I_0$  by ammeter & no-load i/p power  $W_0$  by wattmeter.



→ As the normal rated voltage is applied to primary iron loss will occur in primary. Small amount of copper loss also occurs which can be neglected.

Wattmeter reading  $W_0 =$  iron loss =  $W_i$

No-load current =  $I_0 =$  Ammeter reading

Applied voltage =  $V_1 =$  Voltmeter reading

i/p power,  $W_0 = V_1 I_0 \cos \phi_0$

No-load P.F. ( $\cos \phi_0$ ) =  $\frac{W_0}{V_1 I_0}$

$$\begin{aligned} I_W &= I_0 \cos \phi_0 \\ I_M &= I_0 \sin \phi_0 \end{aligned}$$

$$R_0 = \frac{V_1}{I_W}, X_0 = \frac{V_1}{I_M}$$

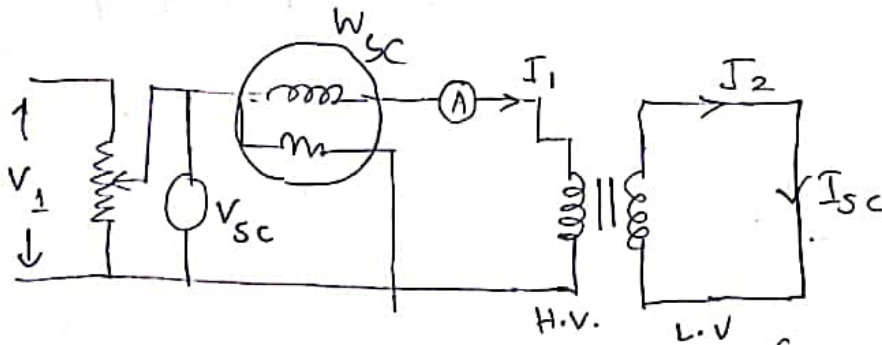


## ② Short circuit test

13

① This test is conducted to determine

- i) Full load copper losses of the transformers
- ii) Series branch parameters  $R_{01}$  &  $X_{01}$



- In this test, the secondary (usually L.V. winding) is short circuited & a variable low voltage is applied to the primary.
- The low i/p voltage is gradually raised till at voltage  $V_{sc}$ , full load current  $I_1$  flows in the primary &  $I_2$  in the secondary also has full load value
- There is no output from the transformer under short-circuit conditions. so i/p power is all loss & entirely copper loss. since  $V_{sc}$  is small iron loss is small & can be neglected,

F.L. cu. loss ( $W_{sc}$ ) = wattmeter reading

Applied voltage ( $V_{sc}$ ) = Voltmeter reading

F.L. primary current ( $I_1$ ) = Ammeter reading

$$W_{sc} = I_1^2 R_1 + I_1^2 R'_2 = I_1^2 R_{01}$$

$$R_{01} = \frac{W_{sc}}{I_1^2}$$

Total impedance,  $Z_{01} = \frac{V_{sc}}{I_1}$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

Short circuit P.F ( $\cos \phi_2$ ) =  $\frac{W_{sc}}{V_{sc} I_1}$

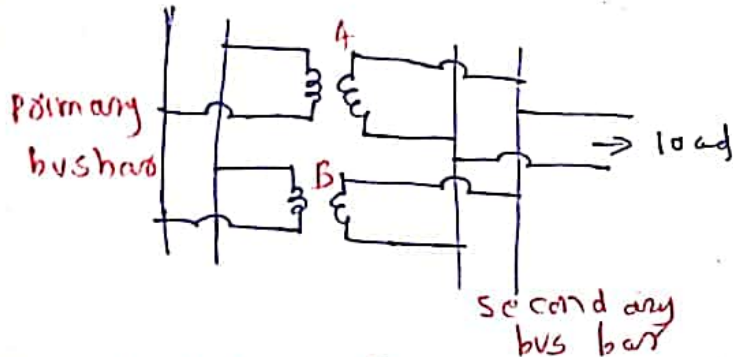
→ s.c. test gives F.L. cu. loss,  $R_{01}$  &  $X_{01}$ .

## Parallel operation of transformers

(15)

(19)

→ Two transformers are said to be in parallel if the primary windings are connected to supply bus bars & secondary windings are connected to load busbars.



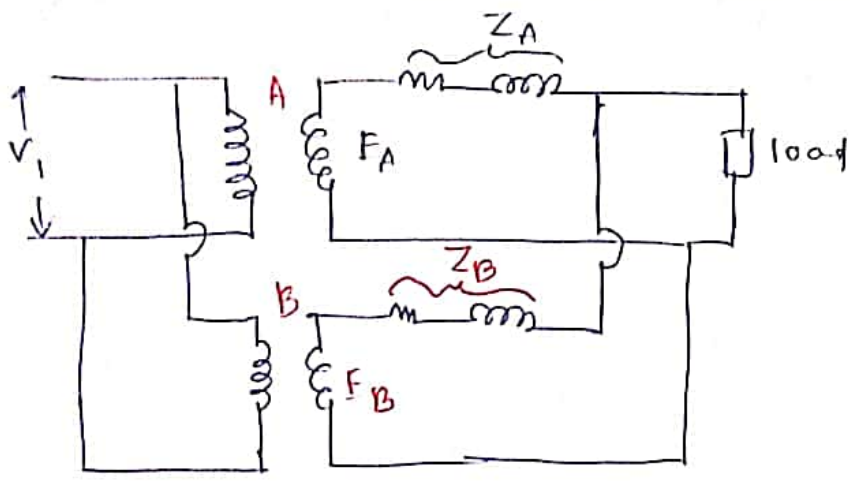
→ Transformer A & B are in parallel.

### Condition for parallel operation of two transformers

- i) Transformer should be properly connected with regard to their polarities.
- ii) The voltage ratings & voltage ratio of the transformer should be same.
- iii) The p.v. or percentage impedances of the T/F should be same.
- iv) The  $\frac{X}{R}$  ratio of the transformers should be same.

### Single phase equal voltage ratio T/F in parallel

→ Two single phase equal voltage ratio transformers A & B in parallel. The secondary E.M.F.s of the two transformers are equal (i.e.  $E_A = E_B = E$ ) because they have the same turns ratio and have their primaries connected to the same supply.



→ if the magnetising current is ignored, the two transformers can be represented by their equivalent circuits referred to secondary. It is clear that the transformers will share total load in the same way as two impedances in parallel.

Let  $Z_A, Z_B$  = Impedances of transformers referred to secondary

$I_A, I_B$  = respective currents

$V_2$  = common terminal voltage

$I$  = total load current

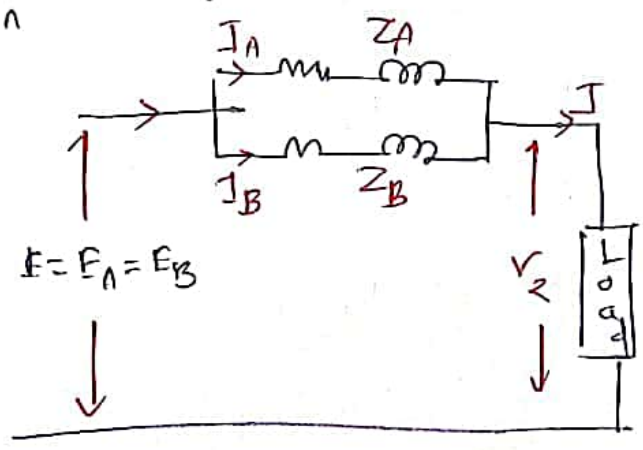
$$I_A + I_B = I$$

$$I_A Z_A = I_B Z_B$$

$$I_A = I_B \frac{Z_B}{Z_A}$$

$$I_B \left( 1 + \frac{Z_B}{Z_A} \right) = I$$

Similarly  $I_A = I \frac{Z_B}{Z_A + Z_B}$





## KVA carried by each transformer

(6)

Let  $S =$  total <sup>load</sup> KVA

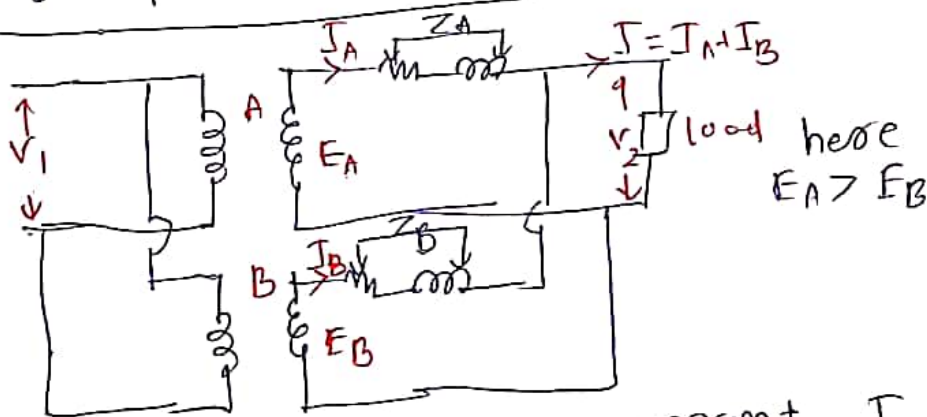
$S_A =$  KVA carried by transformer A

$S_B =$  " " " " B

$$S_A = S \times \frac{Z_B}{Z_A + Z_B}$$

$$S_B = S \times \frac{Z_A}{Z_A + Z_B}$$

## 1- $\phi$ unequal voltage ratio transformers in parallel



At no-load, the circulating current  $I_c$  is

$$I_c = \frac{E_A - E_B}{Z_A + Z_B}$$

$$V_1 = \left[ \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L (Z_A + Z_B)} \right] Z_L$$

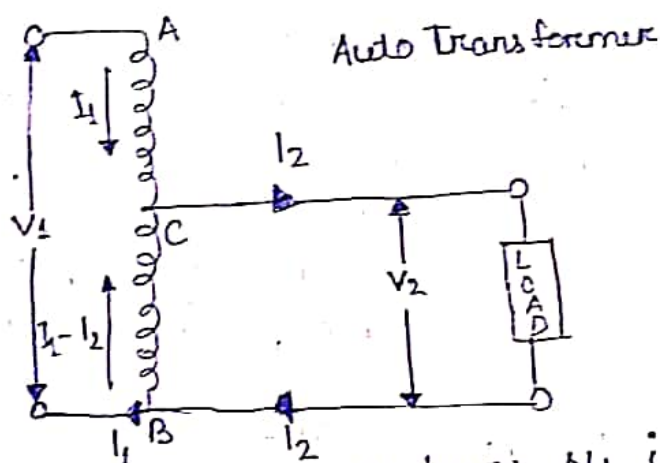
$$V_2 = \frac{E_A Z_B + E_B Z_A}{\frac{Z_A Z_B}{Z_L} + Z_A + Z_B}$$

# AUTO TRANSFORMER

(1)

An auto-transformer is a kind of electrical transformer where primary and secondary shares same common single winding. So basically it's a one winding transformer.

THEORY - In an auto transformer, one single winding is used as primary winding as well as secondary winding. But in two windings transformer two different windings are used for primary and secondary purpose. A circuit diagram of auto transformer is shown below -



The winding AB of total turns  $N_1$  is considered as primary winding. This winding is tapped from point 'c' and the portion BC is considered as secondary. Let's assume the number of turns in between points 'B' and 'c' is  $N_2$ .

If  $V_1$  voltage is applied across the winding i.e. in between 'A' and 'c'.

So voltage per turn in this winding is  $\frac{V_1}{N_1}$

Hence, the voltage across the portion BC of the winding, will be,

$\frac{V_1}{N_1} \times N_2$  and from the figure above, the voltage is  $V_2$ .

$$\text{Hence, } \frac{V_1}{N_1} \times N_2 = V_2$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1} = \text{constant} = K$$

As BC portion of the winding is considered as ② Secondary, it can easily be understood that value of constant 'k' is nothing but turns ratio or voltage ratio of that auto transformer. When load is connected between secondary terminals i.e. between 'B' and 'C' - load current  $I_2$  starts flowing. ~~The~~ The current in the secondary winding or common winding is the difference of  $I_2$  and  $I_1$ .

### Copper Savings in Auto Transformer :-

Now we will discuss the savings of copper in auto transformer compared to conventional two winding transformer.

We know that weight of copper of any winding depends upon its length and cross-sectional area. Again length of conductor in winding is proportional to its number of turns and cross-sectional area varies with rated current.

So weight of copper in winding is directly proportional to product of number of turns and rated current of the winding.

Therefore, weight of copper in the section AC proportional to,

$$(N_1 - N_2) I_1$$

and similarly, weight of copper in the section BC proportional to,

$$N_2 (I_2 - I_1)$$

Hence, total weight of copper in the winding of auto transformer proportional to,

$$\Rightarrow N_1 I_1 - N_2 I_1 + N_2 I_2 - N_2 I_1$$

$$\Rightarrow N_1 I_1 + N_2 I_2 - 2N_2 I_1$$

$$\Rightarrow 2N_1 I_1 - 2N_2 I_1 \quad (\text{since, } N_1 I_1 = N_2 I_2)$$

$$\Rightarrow 2(N_1 I_1 - N_2 I_1)$$



In similar way it can be proved, the weight of <sup>(3)</sup> copper in two winding transformer is proportional to,

$$N_1 I_1 - N_2 I_2$$

$$\Rightarrow 2N_1 I_1 \text{ (since, in a transformer } N_1 I_1 = N_2 I_2)$$

$$N_1 I_1 + N_2 I_2$$

$$\Rightarrow 2N_1 I_1 \text{ (since, in a transformer } N_1 I_1 = N_2 I_2)$$

Let's assume,  $w_a$  and  $w_{tw}$  are weight of copper in auto transformer and two winding transformer respectively,

$$\text{Hence, } \frac{w_a}{w_{tw}} = \frac{2(N_1 I_1 - N_2 I_1)}{2(N_1 I_1)}$$

$$= \frac{N_1 I_1 - N_2 I_1}{N_1 I_1} = 1 - \frac{N_2 I_1}{N_1 I_1}$$

$$= 1 - \frac{N_2}{N_1} = 1 - k$$

$$\therefore w_a = w_{tw} (1 - k)$$

$$\Rightarrow w_a = w_{tw} - k w_{tw}$$

$\therefore$  saving of copper in auto transformer compared to two winding transformer,

$$\Rightarrow w_{tw} - w_a = k w_{tw}$$

Advantages :-

- (1) The leakage flux is less & cu. used is less.
- (2) The auto transformer has high ~~eff~~ efficiency.
- (3) Auto transformer has better voltage regulation.

Uses of Auto transformer

- i) starting of induction motor
- ii) used to regulate the voltage of transmission line known as booster transformer.

## Tap changers

(4)

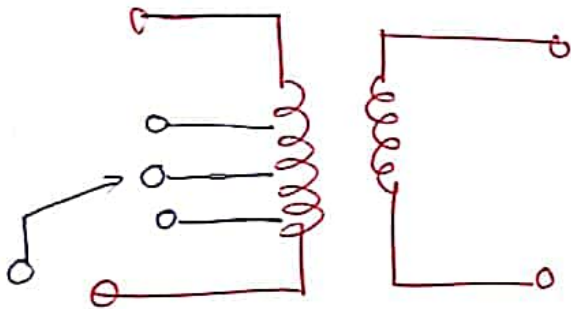
→ A tap changer basically changes the no. of turns of the winding which changes the voltage level.

→ For close control of voltage taps are usually provided on the high voltage winding of the transformer.

It is of 2 types i) off-load tap changing transformer,

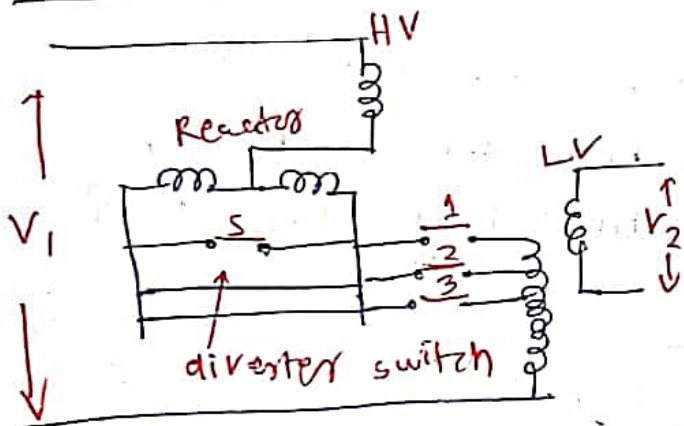
ii) on-load tap changing transformer.

### ① Off-load tap changing transformer



→ In this method, the transformer is disconnected from the main supply when the tap setting is to be changed. The tap setting is to be done manually.

### ② On-load tap changing transformer



→ The tap changing employing a center tapped reactor R. S is the diverter switch, 1, 2, 3 are selector switch. The transformer is in operation with switches 1 & S closed.

→ switch 1 is then opened, & S closed to complete the tap change. It is to be ~~operated~~ noted that the diverter switch operates on load, & no current flows in the selector switches during tap changing.

→ It is to be noted that the diverter switch operates on load & no current flows in the selector switches during tap changing. During the tap change, only half of the reactance which limits the current is connected in the circuit.



# Instrument transformers

①

→ These transformers are used in conjunction with meters for the measurement of high current & high voltage.

→ It is of 2 types i) current transformer (CT)  
ii) Potential transformer (PT) / voltage transformer (VT)

## ① Transformation ratio (R) ÷

It is the ratio of primary phasor to secondary phasor.

$$R = \frac{I_p}{I_s} \Big|_{CT} \quad R = \frac{V_p}{V_s} \Big|_{PT}$$

## ② Nominal ratio (k<sub>n</sub>) ÷

ratio of rated primary phasor to rated secondary phasor.

$$k_n = \frac{\text{rated } I_p}{\text{rated } I_s} \Big|_{CT} \quad k_n = \frac{\text{rated } V_p}{\text{rated } V_s} \Big|_{PT}$$

## ③ Turn's ratio (n) ÷

$$n = \frac{N_s}{N_p} \Big|_{CT} \text{ (step UP TF)} \quad n = \frac{N_p}{N_s} \Big|_{PT} \text{ (step down T/F)}$$

## ④ Ratio correction factor ÷ $\left(\frac{R}{k_n}\right)$ ✓ = $\frac{\text{Actual ratio}}{\text{nominal ratio}}$

⑤ Load = load on T/F will be specified with the name of burden. It will be expressed in V-A.

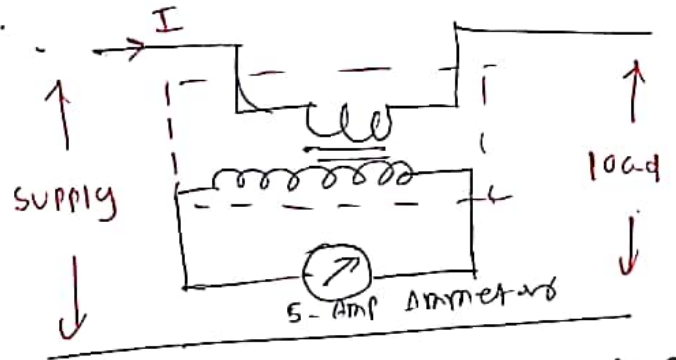
## current transformer (CT)

→ current transformer is used to step down the current to a lower level to make it feasible to be measured by small rating Ammeter.

→ usually 1A will be used for measurement & 5A for protection.

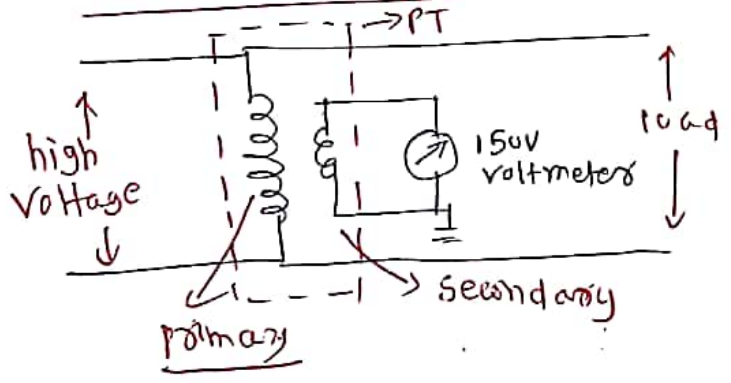
- The conductors constitute a one-turn primary winding. The secondary consists of a large no. of turns of much fine wire wrapped around the core,
- Due to transformer action, the secondary current is transformed to a low value which can be measured by ordinary meters.

$$I_s = I_p \times \frac{N_p}{N_s}$$



The secondary of a CT should ~~be~~ never be left open under any circumstances.

Potential transformers



i) These transformers are extremely accurate - ratio stepdown transformers and are used in conjunction with standard low-voltage range voltmeters (100-20V) which deflection when divided by transformer ratio gives the true voltage on the primary or high voltage side.

(i) In general, they use of shell type & do not differ much from the ordinary two-winding transformers except their power rating is extremely small. Since their secondary windings are required to operate instruments or relays or pilot lights, their rating is usually of 40W to 100W. (3)

(ii) For safety, the secondary is completely insulated from the high voltage primary & in addition, grounded for protection of the operator.